SFE-MODELLING OF DYNAMIC AND STATIC PROPERTIES OF R/C BEAMS IN PROGRESSIVE DAMAGE

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1 Introduction

The application of systematic inspection and maintenance of civil engineering structures has increased enormously during the last decades. Dynamic testing for damage assessment as non-destructive method has attracted growing interest. The applicability of the dynamic investigation does not depend on the building material. It is also possible to detect damage at unobserved locations. Dynamic in situ testing can be a fast method with relatively low cost. The increasing speed of personal computers and data acquisition systems opens new dimensions for dynamic investigations.

The accumulation of damage in a structure causes a change in dynamic properties such as natural frequencies, mode shapes and damping. The occurrence of a crack in a R/C or in a steel structure changes the local stiffness and so the modal properties. In general, the natural frequency decreases, the damping capacity changes up and down and the mode shapes shift slightly.

Only a few studies so far focus on relations between progressive structural damage and changes of dynamic structural behavior. Most of these investigations concentrate on beam structures. Investigations with R/C beams can be found e.g. in Dieterle & Bachmann;1979, Jahn; 1996, Owens et al.; 1999, De Roeck et al.; 1999, for steel beams in Rytter et al.; 2000. Studies of plate structures can be found in Meinhold et al.; 1996; Ebert et al.; 1999.

Structures have time-independent randomly varying material parameters. These properties influence the state of an undamaged structure and the damage evolution. Therefore, the dynamic properties are not only a function of the damage state of a structure, but also of these varying parameters. For understanding and for assessing the actual damage state, it becomes mandatory to investigate the influence of stochastic properties.

In this context static and dynamic experiments with three R/C beams were performed. A Stochastic Finite Element (SFE) Model is developed to investigate numerically the nonlinear behavior of the beams by using correlated random fields for different material and physical properties. Thus, different damage histories are investigated.

2 Changing of dynamic properties due to damage

The equation of motion for a multi degree of freedom structure is given by

$$M \cdot \ddot{x}(t) + C \cdot \dot{x}(t) + K \cdot x(t) = f(t) \qquad (1)$$

where M is the mass matrix, C is the damping matrix and K is the stiffness matrix, f is the load vector. If we consider varying material properties for an undamaged structure, all these matrices are spatially stochastic. Generally, some reference values are given for the production process, e.g. for the strength and for its correlated Young's Modulus. If tests with specimens of the structural material are performed, the first and second statistical moments for the investigated parameters can be determined.

The matrix eigenvalue problem

$$\left(K - \omega_n^2 \cdot \mathrm{M}\right) \cdot \Phi_\mathrm{n} = 0 \tag{2}$$

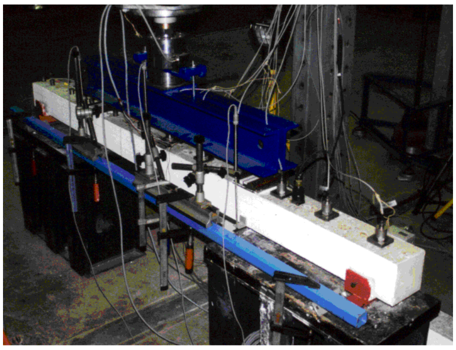
is solved and we get the stochastic natural frequencies ω and the mode shapes as the matrix of eigenvectors Φ.

During the load history the structure is damaged. Damaged zones in a deterministic formulation are the zones, where the stiffness matrix of the originally homogenous structure gets defects. In a stochastic formulation damage occurs, when the homogenous correlated fields of material properties (as integrated in the stiffness matrix) get defects. These defects can be effected by changed material states, e.g. one integration point fails at a limit stress.

Damage for a R/C structure under laboratory conditions is at first crack evolution and the resultant loss of bond between concrete and reinforcement. Other influences on the load carrying or dynamic behavior like thermical, chemical influences (carbonatisation and corrosion), the condition of bearings and other time-dependent material properties may be neglected. The crack evolution decreases the stiffness of the structure. The time and the location of beginning and the further evolution of cracking are stochastic. The stochastic stiffness matrix K is changed while the mass matrix M remains constant. Hence the natural frequencies depend on K only. Within the model of a linear undamped system, the natural frequencies decrease. The mode shapes remain similar, only the location of the nodal lines changes in a small. The damping increases especially at the load levels of crack initiation.

3 Experimental Results

Static and dynamic four point bending tests with three similar R/C beams were performed. The aim of the tests was to determine the changing of modal properties of the beam after each increasing load level. These changes are compared with the static behavior of the structures, especially as stiffness-loss factor of static and dynamic parameters. The dimensions of the beam are $(2.10 \times 0.12 \times 0.115)$m. The beams are reinforced with three longitudinal bars of diameter 6 mm. The beams were permanently simply supported. The Fig. 1 shows the typical test setup and the scheme of instrumentation and excitations for beam B2, only the results of these beams are presented in this paper.



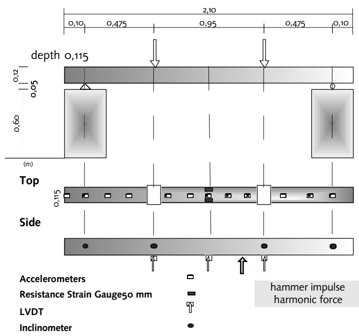


Figure 1. Test setup and scheme of instrumentation, B2

Under static conditions properties displacements, rotations, strains were continuously measured. As dynamic properties acceleration time histories were recorded after each finished load level. A hammer impact load and also harmonic excitation were used as dynamic excitation. The dynamic measurements were always performed on the unloaded structure after removing the steel beam for the load. These dynamic loads result in only small deformations and therefore the measured vibrations are assumed to be linear.

The static load was increased step by step with varying load histories for the beams, but the same load regime for each of the beams. Each load level was repeated three times to get a stable damage state. Fig. 2 shows the load-displacement history with the hysteresis loops and their damage behavior with increasing load. To quantify the damage evolution a stiffness-loss factor D_k is defined in eq. 3. The tangential stiffness k_i of each of the three load processes of each load level is determined. This is done for the three receivers for the displacements (LVDT), (Fig. 3). The stiffness k_0 is the stiffness of the first loading process.

$$D_k = 1 - \frac{k_i}{k_0} \tag{3}$$

In the first load levels a low increasing of the stiffness can be seen, which is effected by conditions of the test setup. At 5 kN a strong crack development happens which leads to a strong decrease in stiffness up to a D_k of 50 %. Then we have a more slowly increasing of stiffness loss up to a level of 72 %. To illustrate the process movie 1 shows the crack evolution for the load levels after Fig. 4.

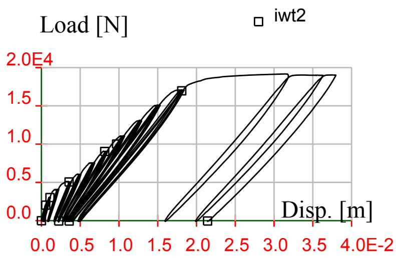


Figure 2. Load-displacement, B2

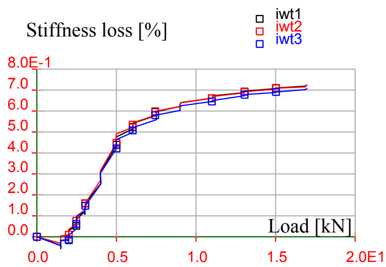
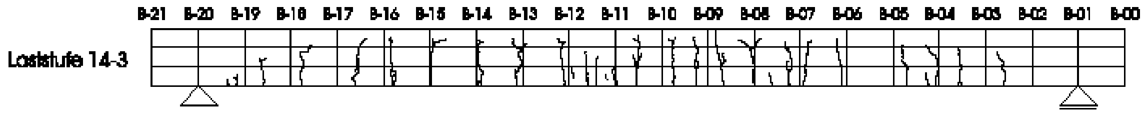


Figure 3. D_k-Load, B2



Movie 1. Crack evolution

Number of load level (3 times)	01. 1-3	02. 1-3	03. 1-3	04. 1-3	05. 1-3	06. 1-3	07. 1-3	08. 1-3	09. 1-3	10. 1-3	11. 1-3	12. 1-3	13. 1-3	14. 1-3
Load level [kN]	1.5	2.0	2.5	3.0	4.0	5.0	6.0	7.5	9.0	11.0	13.0	15.0	17.0	19.0

Figure 4. Load levels

Fig. 5 and 7 show the evolution of the first natural frequency and the modal damping ratio ζ of the first natural frequency of structure B2. Strong decrease of the natural frequency occurs after the load level of 4 and 6 kN. The load- frequency function has the typical tri-linear shape before failure of the structure. The damping ratio increases with crack formation, then decreases to a level which is higher as that at the beginning. Analogous to eq. 3, we define a frequency loss factor D_F (Fig. 6).

$$D_F = 1 - \frac{f_k^2(k)}{f_0^2(k)} \tag{4}$$

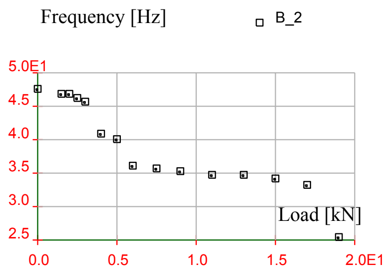


Figure 5. Changing of the first natural frequency, B2

Figure 6 D_F-Load, B2

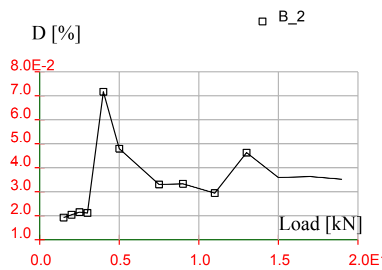


Figure 7 ζ of the first natural frequency, B2

The curves of D_K and D_F show the same qualitative progress. But the quantitative stiffness loss is not the same. The lower stiffness loss, as calculated from the frequencies, is probably a result of the crack closure after unloading.

4 Stochastic Finite Element Model

The beams are modeled three-dimensionally by finite elements. The model consists of 8-node brick elements for the concrete (Fig. 8) and beam elements for the reinforcement. Vertical crack opening zones are defined in a theoretical crack distance for the concrete. These zones (Fig.8) have horizontal three-dimensional spring contact elements. These elements for discrete crack modeling lose their stiffness in beam axis direction at a given force equivalent to a tensile strength (Fig. 9). For the zone between concrete and reinforcement a further contact element is utilized, which considers the failure of bond after cracking of concrete in this area.

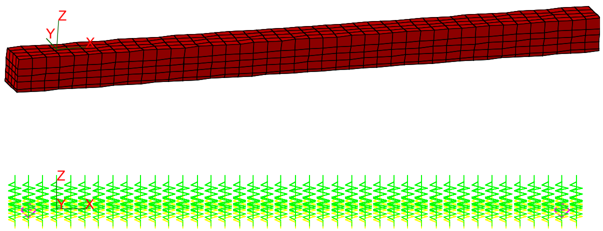


Figure. 8 Brick elements of FE-Model and vertical contact zones (spring elements)

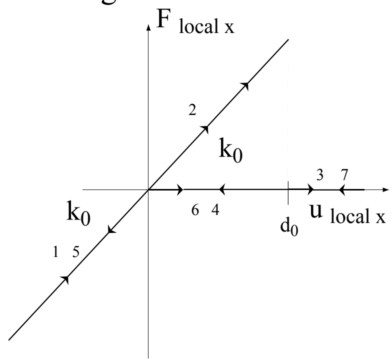


Figure 9. Material law of the spring elements for discrete cracks modeling

The nonlinear model should perform the hysteretic loading- and unloading curves of the different load levels. After unloading a eigenvalue analysis is performed to get the actual eigenvalues of the structure. So both static and dynamic experimental and numerical results of the same FE-model are comparable, a fact which is very rare in the literature.
As mentioned random fields for material and physical properties are used. A random field H(x) is a geometrical multidimensional continuous, stochastic process. The random field is weakly homogenous and weakly isotropy. The discretisized random variables $H(x_i)$; i=1,...,n are correlated through the covariance matrix C_{HH}, which is defined with the aid of the auto-correlation function R_{HH}.

$$C_{HH}(x_i, x_j) = E\left[\left\{H(x_i) - \bar{H}(x_i)\right\}, \left\{H(x_j) - \bar{H}(x_j)\right\}\right];\ i, j = 1, \ldots, n \qquad (6)$$

$$R_{HH} = \sigma^2 \cdot \exp\left(-\frac{\|x - y\|}{L_c}\right) = \sigma^2 \cdot \exp\left(-\frac{\|\xi\|}{L_c}\right) \qquad (7)$$

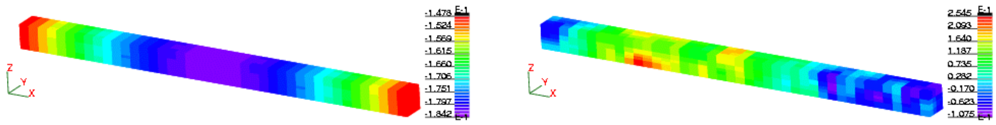
σ^2 is the standard deviation of the random field, L_c is the correlation distance, ξ the distance vector. A non standard normal distributed random field is transformed in the correlated normal space by the Nataf transformation. To create realizations of the random field by Monte Carlo methods the correlated Gaussian random variables are made independent by solving the eigenvalue problem of the covariance matrix. The transformation is

$$h = \Phi c, \qquad (8)$$

with h as a vector of correlated random variables, Φ the matrix of eigenvectors and c the vector of uncorrelated random variables. One important property of Φ is

$$\Phi^T C_{HH} \Phi = \mathrm{diag}(\sigma_i^2), \qquad (9)$$

where σ_i^2 are the variances of the uncorrelated Gaussian random variables. The vector h of the random field is then a linear combination of deterministic eigenvectors Φ_k with random amplitudes. Movie 2 show the first 20 mode shapes and 10 samples of the random field with the parameters after Fig. 10. In many cases the number of random variables can be reduced considerably from n to a much smaller number n*.« n. The quality of the field can be estimated by comparing the considered variances σ_i^2 with the sum of all variances of the uncorrelated random variables vector. Detailed description of the used method for random fields can be found in Brenner; 1995 and SLang; 2000.



Movie 2. First eigenvalue of random field (left) and a sample (pictures from movie)

The computational effort for the Monte Carlo simulation of the nonlinear problem can be reduced by applying the Latin-Hypercube sampling (LHS) technique. This method is particularly useful for the estimation of response variability from a very small number of random samples. A recent study by Novák et al. (2000) showed the excellent applicability of the method for linear and nonlinear random field problems. Fig. 10 presents the parameters of the random field and the samples. The two random fields are full correlated.

Random field for	Type/ Mean Value/ Standard deviation	Correlation length	Used elements	Number of elments/ random variables	Number of used variables for LHS
Stiffness [N/m^2]	normal/3.5e10 /0.2	2.1 m	Brick8, Spring	840; 940	128
Strength [N/m^2]	normal/3.5e6 /0.2	2.1 m	Spring	940	128

Figure 10 Random field parameters

5 Numerical results in comparison with experimental results and their statistics

The SFE computation with LHS is performed with 32 samples. The mean value μ, the standard deviations σ and the detailed functions of all samples are presented in Fig. 11 to Fig. 14. The red line in both figures is the experimental history for the first natural frequency. The experimental and numerical results for the frequency differ after the strong crack formation. The static result of the experiment is located above the mean value function. A contradiction is, that the real beam has higher maximal displacements, but a higher frequency as the model. The reason are probably the complicated processes of bond loss and crack closing and their influence on the dynamic behavior of an unloaded structure as also seen in the stiffness loss factor in Chapter 3. These influences are a topic of current research and are not yet sufficiently represented in the model.

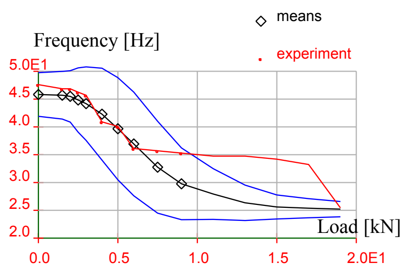


Figure 11. Frequency-Load SFE (μ,σ (blue)) and experiment (B2)

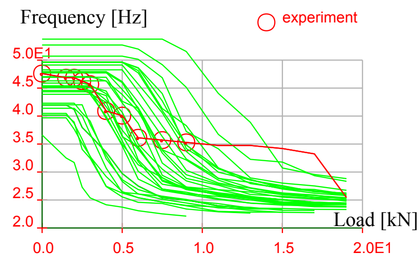


Figure 12 Frequency-Load SFE (LHS, 32 samples) and experiment (B2)

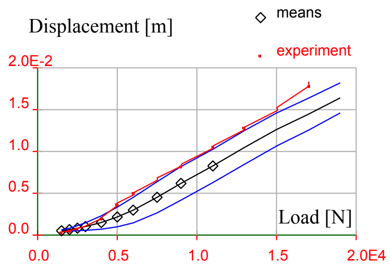


Figure 13 Maximal displacement-Load SFE (μ,σ (blue)) and experiment (B2)

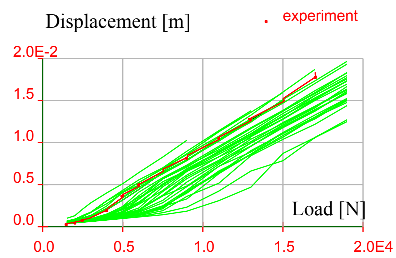


Figure 14 Maximal displacement-Load SFE (LHS, 32 samples) and experiment (B2)

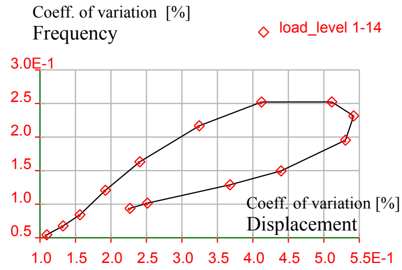


Figure 15 Coefficient of variations of frequency and max. displacements for all load levels

The coefficient of variations increases nearly linearly from the first to the 7th load level. This is an effect of the stochastic crack formation. After that only the variation of the displacements increases. At the higher load levels the variations decrease. The reason is a finished crack evolution, which could also be observed during the experiment. The states of the structure are similar, the influence of stochastic material parameters decreases.

6 Conclusions

The results indicate an influence of stochastic material properties on the damage evolution. By taking into account experimental results a Stochastic Finite Element Model is developed and updated, which allows studying different damage histories. There are still differences between the dynamic and static results, but some phenomena are well described qualitatively. The use of random fields allows to perform more realistic computations, although the parameters of such fields, especially the correlations, are quite uncertain. Advanced Monte-Carlo simulation techniques such Latin Hypercube Sampling can lead to a substantial reduction of numerical efforts of the nonlinear computations. So the different histories of structures with the same design parameters can be investigated. At the end this knowledge is the basis for further development of dynamic testing procedures and the assessment of its profits.

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