

TOWARDS OPTIMAL DESIGNING OF THIN ELASTIC PLATES WITH A SPECIFIED FREE OSCILLATIONS FREQUENCY

Prof. Tanko Ganev, Dr. Eng.,

Department of Structural Mechanics, University of Architecture, Civil Engineering and Geodesy,
1 Chr. Smirnenski blvd, 1421 Sofia, Bulgaria,

Milen Marinov, PhD. Eng.,

Department of Mechanics, Higher Military School of Transport,
158 Geo Milev blvd, 1574 Sofia, Bulgaria.

Introduction

Structures subjected to complex dynamic effects with a known resonance range are widely applied in practice. One of the most important stages of their designing is the resonance state. On the one hand the resonance phenomena with ordinary materials sharply increase the dynamic load. From the point of view of strength measures should be taken to remove them for example by widening the zones of operation without resonance which is necessary especially for building structures.

On the other hand however many elements and structures (mostly machine-building) operate in resonance state as a main operation mode that's why it is desirable to draw the natural frequencies nearer to those of the dynamic effects.

Having in mind the main and increasingly important requirement for economy of contemporary structures it follows that both problems presented above are optimization dynamic problems. They can be solved by such a distribution of structure parameters called design variables where its natural frequencies take a state determined in relation to the dynamic effects frequencies.

Regardless of its practical significance this dynamic optimization problems class has not been fully examined yet [Áîãîîëîâ, Ñèîñîí, 1983]. Setting the condition that the structure operates in a zone without resonance the optimization problems are most often solved for natural frequency spectrum control [Arman, 1977; Olhoff, 1982; Áàíè÷óê, Èâàíîâ, Øäðàíþê, 1989] or for forced oscillations according to the stressed and deformed state criteria [Ãðéíâ, Ôèèèñîâ, 1975; Haug, Arora, 1983].

The paper presents a possible approach to solve the problem of thin elastic plates design with minimum weight in resonance operating modes. An effective and sufficiently common methods are worked out on the base of Wilder's energy resonance method, the method of finite elements for dynamic research and the methods of parameter optimization.

Formulation of the Optimization Problem

The primary design of homogeneous thin elastic plate is for a constant thickness $t^{(0)}$ and one of its natural frequencies is $\omega_j^{(0)}$. The plate takes the area Ω with a boundary $\partial\Omega$. Its weight is $G^{(0)} = \rho g A t^{(0)}$ where: ρ is a density of the material which the plate is made of; g - a gravitational acceleration; A - a area limited by the boundary $\partial\Omega$.

Harmonious dynamic effect with frequency θ is applied to the plate. The plate should reach the resonance state of $\omega_j = \theta$ and continue its operation in that mode.

To satisfy the non-linear strength and stiffness limitations, that is to take into account the influence of the dynamic effect magnitude, a minimal admissible plate thickness t^L is assigned. In this way construction requirements to the plate are taken into account. To apply the theory to thin elastic

plates a maximal admissible plate thickness t^U is assigned too. This limitation is a part of technology requirements as well.

Hence the variation area of the plate thickness $t(x_1, x_2)$ taken to be a design variable, is

$$(1) \quad D = \left\{ t: t^L \leq t(x_1, x_2) \leq t^U, \quad 0 < t^L < t^{(0)}, \quad \omega_j = \theta \right\}.$$

that is the area (1) is the one of the admissible plate designs.

A plate design with minimal weight is searched

$$(2) \quad G^* = \min_{t \in D} \rho g \int_{\Omega} t(x_1, x_2) dx_1 dx_2.$$

which is its optimal design.

The problem can be conventionally marked by $\omega_j \rightarrow \theta \rightarrow \min G$.

Solution of the Optimization Problem

From the area of admissible designs (1) it follows that with changing the natural plate thickness its natural frequencies cannot take arbitrary values and they change in the following intervals [Kozlov, 1979]

$$(3) \quad \omega_1 \in [\omega_1^-, \omega_1^+], \quad \omega_2 \in [\omega_2^-, \omega_2^+], \quad \dots, \quad \omega_j \in [\omega_j^-, \omega_j^+], \quad \dots, \quad j = 1, 2, \dots, \infty,$$

where ω_j^- and ω_j^+ are respectively the smallest and the biggest possible values of each natural frequencies ω_j .

Hence to arise the phenomenon of resonance the dynamic effects frequency must come into some of these intervals, e.g. into the interval $[\omega_j^-, \omega_j^+]$. With assigned primary plate design and dynamic effects frequency θ the following cases are possible (it is supposed that the intervals (3) don't overlap):

1. $\theta < \omega_j^-$ - a resonance state is not reached as $\omega_j \neq \theta$. With optimization the plate thickness $t(x_1, x_2)$ takes the value of t^L which the minimal plate weight corresponds to¹⁾.
2. $\theta > \omega_j^+$ - in that case a resonance state is not reached. With optimization the thickness takes the value of t^U which the maximal weight corresponds to.
3. $\theta = \omega_j^{(0)}$ - the plate is in resonance state and with optimization it is enough only to minimize its weight preserving its resonance natural frequency.
4. $\theta \neq \omega_j^{(0)}$, but $\theta \in [\omega_j^-, \omega_j^+]$ - the plate is not in a resonance state but it can be reached. Hence at first this state must be reached, that is $\omega_j = \theta$, after that it is to minimize the plate weight preserving the resonance natural frequency.

If some of the intervals (3) overlap the resonance state can be reached. It should be which one or which ones of its natural frequencies may possibly overlap with the specified dynamic effect frequencies. Hence the last case should be applied once or several times. After that it is necessary to compare the obtained optimal plate designs appraising which one is the most economic.

The further statement is connected with the examination of the last most common case.

The analysis of the plate dynamic behaviour in the kind of a forced resonance oscillation problem is complex for many reasons. One of them is that to the condition of a constant resonance

¹⁾ Further abstraction of the material is impossible and adding some to any point of the plate will not decrease its natural frequency in all cases.

frequency $\omega_j = \theta$ is difficult to be satisfied because the optimization of the plate will most probably bring it out of the resonance state.

These difficulties can be successfully overcome by applying Wilder's energy resonance method. Because the resistance forces with the elastic structures doing small oscillations are comparatively small, the form of the force resonance oscillation is very close to the resonance natural form of oscillation, that is

$$(4) \quad \{v(\{x\}, t)\} = a \{\varphi(\{x\})\} \cos(\theta t + \delta),$$

where: $\{v\}$ is a vector of generalised displacement; $\{x\}$ - a vector of space coordinates; $\{\varphi\}$ - a resonance natural form; a - an amplitude of forced resonance oscillations; t - a current moment of time.

Hence by means of effect over the resonance natural form by plate thickness changing the desirable plate behaviour is achieved in resonance state.

This method gives a possibility to reduce physically and geometrically linear dynamic problem to the problem of plate natural frequencies and oscillation form [Áîñîîîîîî, Ñèîñîî, 1983]. For that aim it is necessary to present the optimal state criterion in the kind of

$$(5) \quad J = J(\{\varphi\}, t(x_1, x_2)).$$

The digitized model of a thin elastic plate is presented by N number of finite elements with a constant thickness and is with n degrees of freedom. The components of vector t_r ($r = 1, 2, \dots, N$) must belong to the area of admissible designs (1).

The free oscillation equations of a plate linear digitalized model can be written in the way of

$$(6) \quad [K(\{t\})] \{\varphi_j\} - \lambda_j [M(\{t\})] \{\varphi_j\} = \{0\}, \quad j = 1, 2, \dots, n,$$

where: $[K]$ and $[M]$ are respectively matrixes of plate stiffness and equivalent masses; $\{\varphi_j\}$ - an eigenvector that corresponds to the natural frequency ω_j ; $\lambda_j = \omega_j^2(\{t\})$ - an eigenvalue.

The condition that

$$(7) \quad \{\varphi_j\}^T [M] \{\varphi_j\} = \delta_{ij}, \quad i, j = 1, 2, \dots, n$$

is satisfied for the system of eigenvectors $\{\varphi_j\}$ where δ_{ij} is Kronecker's symbol ($\delta_{ij} = 1$ with $i = j$, $\delta_{ij} = 0$ with $i \neq j$).

The presented methods contain two main points: reaching the specified plate natural frequency θ value of $\omega_j(\{t\})$ and minimizing its weight $G(\{t\})$.

To find a design with frequency $\omega_j(\{t\}) = \theta$ from the specified primary plate design the method of direct search and recurrent dependency is used

$$(8) \quad \{t\}^{(k+1)} = \{t\}^{(k)} \pm \{\delta t\}^{(k)}, \quad k = 0, 1, 2, \dots,$$

where: $\{t\}^{(0)}$ - is a primary vector of finite elements thickness; $\{\delta t\}^{(k)}$ - a vector of thickness changing; (k) - a number of iteration.

The vector components $\{\delta t\}$ of each iteration are taken to be proportional to the corresponding vector-gradient components $\nabla \lambda_j(\{\delta t\})$ of the eigenvalue λ_j which are of the following kind

$$(9) \quad \frac{\partial \lambda_j}{\partial t_r} = \{\varphi_j\}^T \frac{\partial [K]}{\partial t_r} \{\varphi_j\} - \lambda_j \{\varphi_j\}^T \frac{\partial [M]}{\partial t_r} \{\varphi_j\}, \quad r = 1, 2, \dots, N.$$

The dependencies (9) are valid if the equations of free oscillations have as solutions a single eigenvalue.

The iteration process stops when the condition

$$(10) \quad \left| \frac{\omega_j(\{t\}) - \theta}{\theta} \right| 100\% \leq \Delta \%$$

is satisfied, where Δ is the criterion of precision specified by the designer.

The plate has reached the resonance state and the form of forced resonance oscillations is very close to the resonance natural form $\{\varphi_j\}$.

After that the maximum of the optimal state criterion is searched

$$(11) \quad J = \delta G(\{\delta t\}) = \sum_{r=1}^N \delta G_r(\delta t_r) = \{g\}^T \{\delta t\},$$

where: δG_r is a weight change of r -finite element; $\{g\} = \{g_1, \dots, g_r, \dots, g_N\}^T$ - a vector of weight coefficients of finite elements ($g_r = \rho g A_r$, where A_r is the area of r -finite element). It expresses the plate weight change and can be easily presented in the kind of

$$(12) \quad J = + \sum_{\substack{r=1 \\ r \neq s}}^N g_r \delta t_r - \frac{g_s}{\frac{\partial \lambda_j}{\partial t_s}} \sum_{\substack{r=1 \\ r \neq s}}^N \delta t_r \frac{\partial \lambda_j}{\partial t_r} = - \{\delta t\}^T \nabla J(\{\delta t\}),$$

where $\nabla J(\{\delta t\})$ is the optimal state criterion vector-gradient in regards to the vector components $\{t\}$. The optimal state criterion (11) depends directly on the resonance natural frequency ω_j and the resonance natural form $\{\varphi_j\}$.

The dependence (8) is used again as $N-1$ numbers of vector components $\{\delta t\}$ take values proportional to the respective components of the vector-gradient $\nabla J(\{\delta t\})$. The component δt_s of the last finite element S is determined by the condition

$$(13) \quad \delta \lambda_j(\{\delta t\}) = 0,$$

where $\delta \lambda_j$ is the eigenvalue λ_j , change obtained after a linear approximation of non-linear dependency $\lambda_j(\{t\})$.

The optimization process stops when

$$(14) \quad \left\| \nabla J(\{\delta t\}) \right\| \leq \varepsilon,$$

where ε is the criterion of precision specified by the designer too.

In both iteration processes if the limitations in (1) are violated for any component of the vector $\{t\}$ the thickness takes respectively the value of

$$(15) \quad t_r = t^L \quad \text{or} \quad t_r = t^U$$



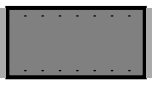
and after that the processes continue.

Numerical Results

The primary design of optimized square plate is with sizes $L = 1 \text{ m}$ and thickness $t^{(0)} = 0,05 \text{ m}$. The plate is made of steel with a module of linear elastic deformations $E = 2,1 \cdot 10^{11} \text{ N/m}^2$, material density $\rho = 7,8 \cdot 10^3 \text{ kg/m}^3$ and Poisson's coefficient $\nu = 0,3$. The plate weight is $G^{(0)} = 3,826 \cdot 10^3 \text{ N}$. A harmonious dynamic load with a frequency θ is evenly distributed over the plate. Limitations

$t^L = 0,03 \text{ m}$ and $t^U = 0,07 \text{ m}$, criteria of preciseness $\Delta = 0,5 \%$ and $\varepsilon = 0,0005$ and the requirement of plate operation in resonance state with $\omega_1 = \theta$ are specified.

The plate is optimized by the worked out programme module with three ways of its supporting - restrained, freely supported, restrained along two opposite sides and freely supported along the other two sides. Only one fourth of the plate is examined for the symmetry of geometry, boundary conditions and mass distribution. It is presented with sixty-four finite elements with equal sizes. An incompatible square plate finite element with twelve degrees of freedom is used. To reduce the number of variable thickness

Plate	θ, s^{-1}	G^*, N	$\frac{G^f - G^*}{G^f}, \%$
	$2,7 \cdot 10^3$	$2,911 \cdot 10^3$	23,9
	$1,5 \cdot 10^3$	$3,013 \cdot 10^3$	21,2
	$2,2 \cdot 10^3$	$2,856 \cdot 10^3$	25,3

of finite elements the last ones are grouped into four as one thickness t_r ($r = 1, 2, \dots, 16$) is common for the whole group.

According to the boundary conditions some data about the material economy of the whole plate is given in the table. The comparison is made to the weight of G^f of the same plate with frequency θ but with a constant thickness.

The optimal plate designs with the three ways of its support are shown in fig. 1-3 and its resonance natural forms respectively in fig. 4-6.

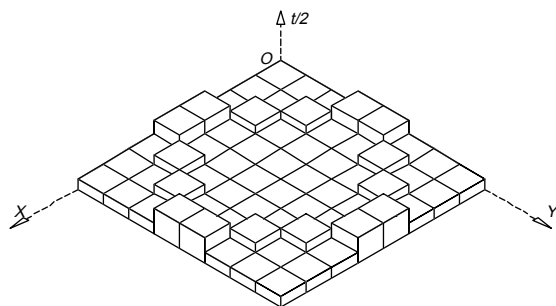


Fig. 1

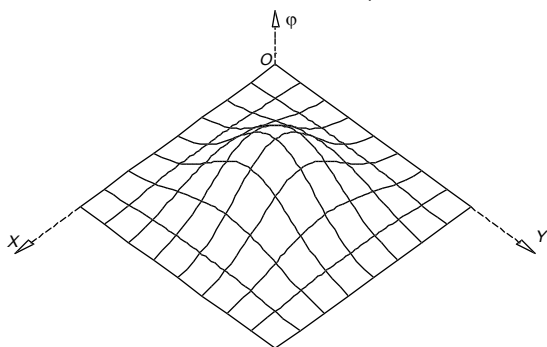


Fig. 4

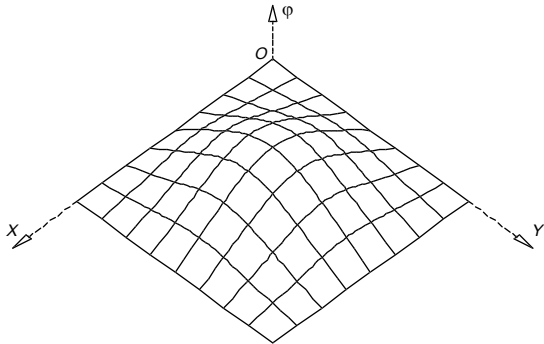
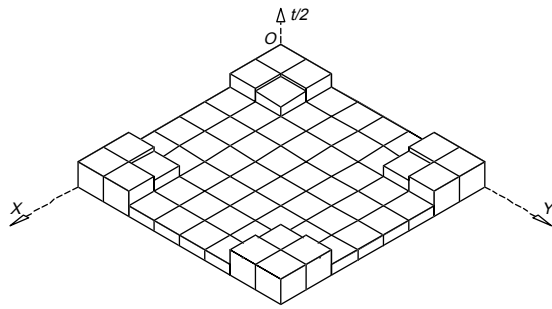


Fig. 2

Fig. 5

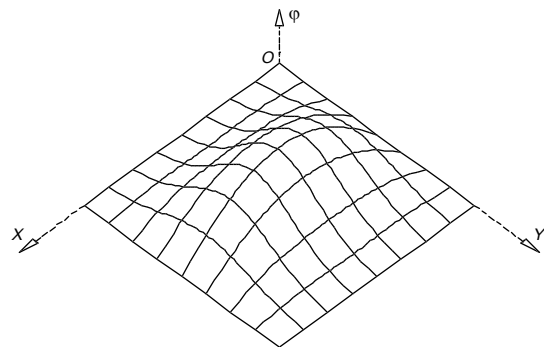
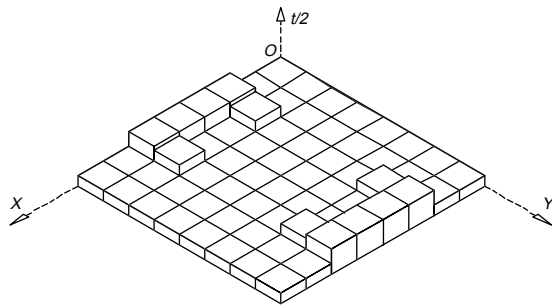


Fig. 3

Fig. 6

Conclusion

The obtained optimal plate designs give a possibility to make the following most important conclusions:

1. A part of the plate material is concentrated in a definite zones with significantly greater bending stiffness, that is a plate with uneven thickness is adapted better to oscillation conditions.
2. To make better the preciseion of the analysis problem it is necessary to compare the finite elements network system as at the same time by their grouping the number of design variables should be reduced aiming at reasonable technology plate making.
3. With the introduced limitations of plate thickness the economy of the material exceeds 20% as over 75% of the plate area are with the specified minimal thickness t^L .

4. Making of optimal plate designs in practice is economically justifiable because the jump-like thickness change facilitates its construction, e.g. by putting additional linings, strengthening ribs or smooth turning of machine elements.

References

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