DISCRETE-CONTINUAL FINITE ELEMENT METHOD OF ANALYSIS FOR THREE-DIMENSIONAL CURVILINEAR STRUCTURES

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Invariability of physical and geometrical parameters in one dimension exists in various problems of analysis of three-dimensional curvilinear structures and corresponding mathematical models. We should mention here in particular such objects as beams, extensional buildings, pipelines, rails, dams and others. Analytical solution is apparently preferable in all aspects for qualitative analysis of calculation data. It allows investigator to consider boundary effects when some components of solution are rapidly varying functions. Due to the abrupt decrease inside of mesh elements in many cases their rate of change can't be adequately considered by conventional numerical methods while analytics enables study. Another feature of the proposing method is the absence of limitations on lengths of structures. Hence it appears that in this context discrete-continual finite element method considering in the distinctive paper is peculiarly relevant. Semianalytical formulations are contemporary mathematical models which currently becoming available for computer realization.

1. FORMULATIONS OF PROBLEM OF ELASTIC THEORY IN CURVILINAR COORDINATE SYSTEMS

1.1. Conventional formulation of the problem in three-dimensional ring coordinate system.

Let us introduce so-called three-dimensional ring coordinate system (t,z,s) especially convenient for the wide range of considering problems (Figure 1). Often and often we have $H = \text{const}$. Therefore this case is of paramount importance.

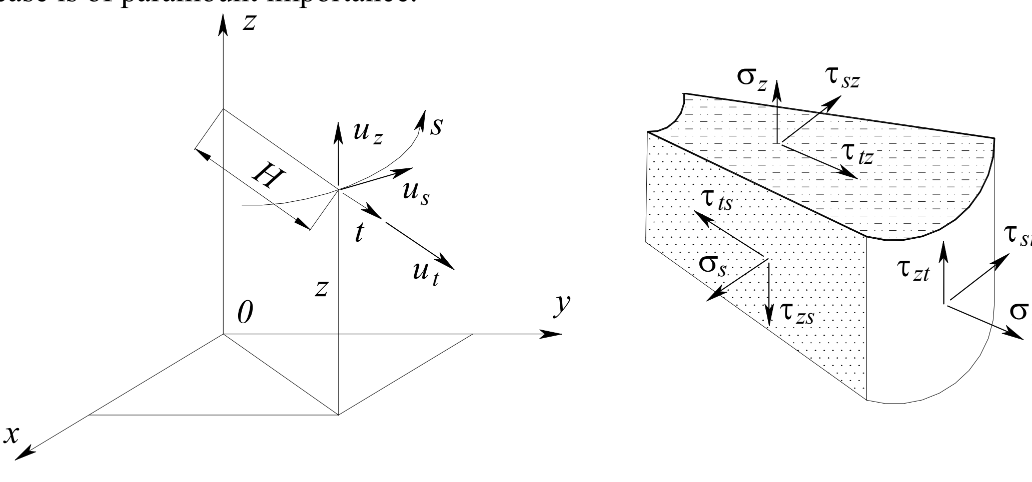

Figure 1.

Here Let u_t, u_z, u_s are displacement components; $\sigma_t, \sigma_z, \sigma_s, \tau_{tz}, \tau_{zs}, \tau_{ts}$ are stress components; $\varepsilon_t, \varepsilon_z, \varepsilon_s, \gamma_{tz}, \gamma_{zs}, \gamma_{ts}$ are strain components; F_t, F_z, F_s are components of body forces.

Considering displacements as basic unknowns we can get conventional formulation of problem of elastic theory in three-dimensional ring coordinate system in the form

$$L\bar{u} = \bar{F}, \quad (t,z,s) \in \Omega; \quad T\bar{u} = \bar{f}, \quad (t,z,s) \in \partial\Omega, \tag{1.1}$$

where Ω is the domain occupied by structure; L is the operator defining conditions in the domain; T is the operator defining conditions at the domain boundary $\partial\Omega$; f_t, f_z, f_s are components of boundary traction vector; ν_t, ν_z, ν_s are components of the unit normal vector on domain boundary,

$$\bar{f} = [\, f_t \quad f_z \quad f_s \,]^T; \quad \bar{\nu} = [\, \nu_t \quad \nu_z \quad \nu_s \,]^T. \tag{1.2}$$

1.2. Operational formulation of the problem in three-dimensional ring coordinate system.
Operational formulation of the problem has the form

$$\mathcal{L}\,\bar{u} = \bar{F}; \quad \mathcal{L} = \theta L\bar{u} + \delta_b T\bar{u}; \quad F_i = \theta F_i + \delta_\Gamma f_i, \tag{1.3}$$

where $\theta(x)$ is the characteristic function of domain; δ_b is the delta function of domain boundary. In the case of isotropic medium operator of considering problem in displacements has the form

$$\mathcal{L} = \{\mathcal{l}_{ij}\}_{i,j=1,2,3}; \tag{1.4}$$

$$\mathcal{l}_{11} = -\partial_t(\bar{\lambda} + 2\bar{\mu})\partial_t - \partial_t \frac{\bar{\lambda}}{H+t} - \frac{2\bar{\mu}}{H+t}\partial_t + \frac{2\bar{\mu}}{(H+t)^2} - \partial_z\bar{\mu}\partial_z - \frac{H^2}{H+t}\partial_s \frac{\bar{\mu}}{(H+t)}\partial_s; \tag{1.5}$$

$$\mathcal{l}_{12} = -\partial_t\bar{\lambda}\partial_z - \partial_z\bar{\mu}\partial_t; \quad \mathcal{l}_{13} = -\partial_t \frac{\bar{\lambda}H}{H+t}\partial_s + \frac{2\bar{\mu}H}{(H+t)^2}\partial_s - \frac{H}{H+t}\partial_s\bar{\mu}\partial_t + \frac{H}{H+t}\partial_s \frac{\bar{\mu}}{H+t}; \tag{1.6}$$

$$\mathcal{l}_{21} = -\partial_z\bar{\lambda}\partial_t - \partial_z \frac{\bar{\lambda}}{H+t} - \frac{\bar{\mu}}{H+t}\partial_z - \partial_t\bar{\mu}\partial_z; \quad \mathcal{l}_{23} = -\partial_z \frac{\bar{\lambda}H}{H+t}\partial_s - \frac{H}{H+t}\partial_s\bar{\mu}\partial_z; \tag{1.7}$$

$$\mathcal{l}_{22} = -\partial_z(\bar{\lambda} + 2\bar{\mu})\partial_z - \partial_t\bar{\mu}\partial_t - \frac{\bar{\mu}}{H+t}\partial_t - \frac{H^2}{H+t}\partial_s \frac{\bar{\mu}}{H+t}\partial_s; \quad \mathcal{l}_{32} = -\frac{H}{H+t}\partial_s\bar{\lambda}\partial_z - \partial_z \frac{\bar{\mu}H}{H+t}\partial_s; \tag{1.8}$$

$$\mathcal{l}_{31} = -\frac{H}{H+t}\partial_s\bar{\lambda}\partial_t - \frac{H}{H+t}\partial_s \frac{\bar{\lambda} + 2\bar{\mu}}{H+t} - \partial_t \frac{\bar{\mu}H}{H+t}\partial_s - \frac{2\bar{\mu}H}{(H+t)^2}\partial_s; \tag{1.9}$$

$$\mathcal{l}_{33} = -\frac{H^2}{H+t}\partial_s \frac{\bar{\lambda} + 2\bar{\mu}}{H+t}\partial_s - \partial_t\bar{\mu}\partial_t + \partial_t \frac{\bar{\mu}}{H+t} - \frac{2\bar{\mu}}{H+t}\partial_t + \frac{2\bar{\mu}}{(H+t)^2} - \partial_z\bar{\mu}\partial_z. \tag{1.10}$$

Here we used the following notation

$$\bar{\lambda} = \theta\lambda; \quad \bar{\mu} = \theta\mu. \tag{1.11}$$

Let Os be coordinate axis with invariability of physical and geometrical parameters of structure (basic direction). We have

$$\mathcal{L} = -\mathcal{L}_{vv}\partial_s^2 + \tilde{\mathcal{L}}_{uv}\partial_s + \mathcal{L}_{uu}; \quad \tilde{\mathcal{L}}_{uv} = \mathcal{L}_{uv} - \mathcal{L}_{uv}^*; \quad \mathcal{L}_{vu} = \mathcal{L}_{uv}^*; \quad \tilde{\mathcal{L}}_{uv}^* = -\tilde{\mathcal{L}}_{uv}. \tag{1.12}$$

It is necessary to clarify here that $\mathcal{L}_{uv}^*$ is the differential operator conjugate to $\mathcal{L}_{uv}$, $\tilde{\mathcal{L}}_{uv}$ is the skew-symmetric operator. Let

$$\bar{v} = [v_t \;\; v_z \;\; v_s]^T = [\partial_s u_t \;\; \partial_s u_z \;\; \partial_s u_s]^T = \partial_s\bar{u} = \bar{u}'; \quad \bar{v}' = [\partial_s v_t \;\; \partial_s v_z \;\; \partial_s v_s]^T = \partial_s\bar{v}. \tag{1.13}$$

If we combine (1.3) with (1.12) and (1.13) we obtain the following set of differential equations:

$$\bar{U}' = \tilde{\mathcal{L}}\bar{U} + \bar{\bar{F}}; \quad \bar{U} = \begin{bmatrix} \bar{u} \\ \bar{v} \end{bmatrix}; \quad \bar{U}' = \partial_s\bar{U} = \begin{bmatrix} \partial_s\bar{u} \\ \partial_s\bar{v} \end{bmatrix} = \begin{bmatrix} \bar{u}' \\ \bar{v}' \end{bmatrix}; \quad \tilde{\mathcal{L}} = \begin{bmatrix} 0 & E \\ \tilde{\mathcal{L}}_{vv}^{-1}\mathcal{L}_{uu} & \mathcal{L}_{vv}^{-1}\tilde{\mathcal{L}}_{uv} \end{bmatrix}; \quad \bar{\bar{F}} = -\begin{bmatrix} 0 \\ \mathcal{L}_{vv}^{-1}\bar{F} \end{bmatrix}. \tag{1.14}$$

1.3. Variational formulation of the problem in three-dimensional ring coordinate system.
Solution of the considering problem is the extreme point (function) of functional

$$\Phi(\bar{u}, \bar{v}) = 0.5 \cdot [(\mathcal{L}_{vv}\bar{v}, \bar{v}) + 2(\tilde{\mathcal{L}}_{uv}\bar{v}, \bar{u}) + (\mathcal{L}_{uu}\bar{u}, \bar{u})] - (\bar{F}, \bar{u}). \tag{1.15}$$

with condition (1.13). Otherwise we have

$$\Phi(\bar{U}) = 0.5 \cdot (\tilde{\tilde{\mathcal{L}}}\,\bar{U}, \bar{U}) - (\bar{\bar{F}}, \bar{U}); \tag{1.16}$$

$$\tilde{\mathcal{L}}_{vu} = \tilde{\mathcal{L}}_{uv}^* = -\tilde{\mathcal{L}}_{vu}; \quad \tilde{\tilde{\mathcal{L}}} = \begin{bmatrix} \mathcal{L}_{uu} & \tilde{\mathcal{L}}_{uv} \\ \tilde{\mathcal{L}}_{uv}^* & \mathcal{L}_{vv} \end{bmatrix} = \begin{bmatrix} \mathcal{L}_{uu} & \tilde{\mathcal{L}}_{uv} \\ \tilde{\mathcal{L}}_{vu} & \mathcal{L}_{vv} \end{bmatrix}; \quad \bar{\bar{F}} = \begin{bmatrix} \bar{F} \\ 0 \end{bmatrix}. \tag{1.17}$$

2. METHOD OF EXTENDED DOMAIN

In accordance with distinctive approach the given domain Ω is embordered by extended one ω of arbitrary shape, particularly elementary (for instance, parallelepiped, cylinder and others). Cross-section of extended domain perpendicular to basic direction is optimally approximated by mesh, topologically equivalent to rectangular. Corresponding key features includes regular numeration of nodes and therefore convenient mathematical formulas, effective computational schemes and algorithms, simple data processing and so on.

3. DISCRETE-CONTINUAL FINITE ELEMENT METHOD

3.1. Discrete-continual design model. Special discrete-continual design model is introduced for three-dimensional problems. It presupposes mesh approximation for tentatively cross directions (Ot, Oz) of extended domain while in the basic direction (Os) problem remains continual. Thus extended domain ω is divided into curvilinear discrete-continual finite elements ω_{ij} (Figure 2).

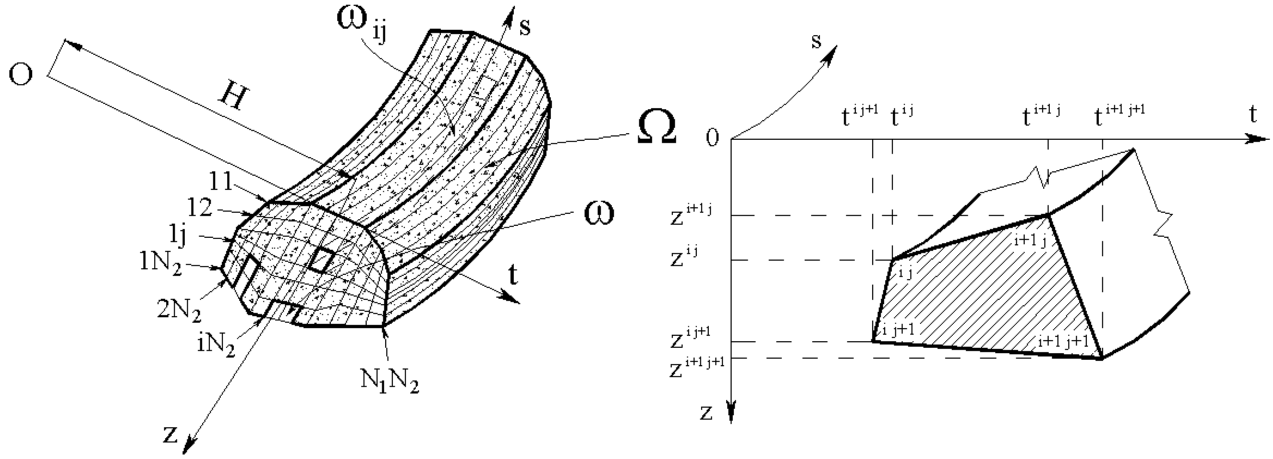

Figure 2.

3.2. Element coordinate system. Local coordinate system is introduced in arbitrary cross-section of element, Renumbering of nodes in cross-section of element is performed (Figure 3).

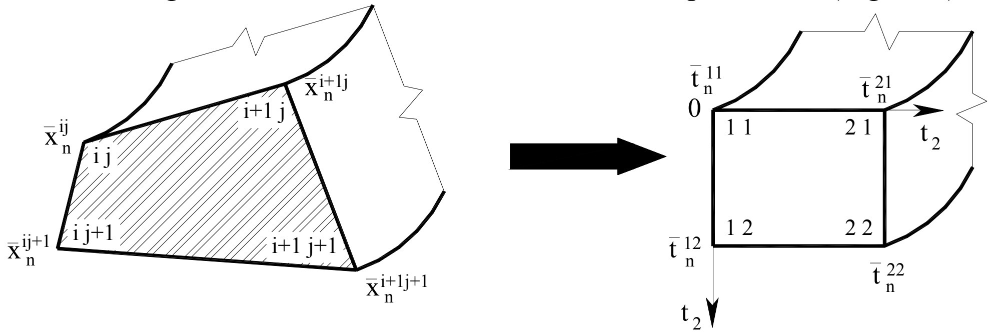

Figure 3.

3.3. Approximation of nodal unknown functions. Basic nodal unknown functions are displacement components u_1, u_2, u_3 and their derivatives v_1, v_2, v_3 with respect to s. Element vector of nodal unknowns has the form

$$\bar{U}^{ij} = \bar{U}^{ij}(s) = [(\bar{U}_n^{11})^T\ (\bar{U}_n^{21})^T\ (\bar{U}_n^{12})^T\ (\bar{U}_n^{22})^T]^T, \qquad (3.1)$$

where

$$\bar{U}_n^{pq} = \bar{U}_n^{pq}(s) = [(\bar{u}_n^{pq})^T\ \ (\bar{v}_n^{pq})^T]^T;\ \bar{u}_n^{pq} = \bar{u}_n^{pq}(s) = [u_t^{pq}\ \ u_z^{pq}\ \ u_s^{pq}]^T;\ \bar{v}_n^{pq} = \bar{v}_n^{pq}(s) = [v_t^{pq}\ \ v_z^{pq}\ \ v_s^{pq}]^T. \qquad (3.2)$$

Within the cross-section of the element we use bilinear approximation of unknowns.

3.4. Construction of element nodal load vector. The element nodal load vector has the form

$$\overline{R}^{ij}=\overline{R}^{ij}(s)=[(\overline{R}_n^{11})^T\ (\overline{R}_n^{21})^T\ (\overline{R}_n^{12})^T\ (\overline{R}_n^{22})^T]^T;\ \ \overline{R}_n^{pq}=\overline{R}_n^{pq}(s)=[(\overline{R}_{u,n}^{pq})^T\ (\overline{R}_{v,n}^{pq})^T]^T;\ \ \overline{R}_{v,n}^{pq}=0. \quad (3.3)$$

3.5. Construction of stiffness matrix of discrete-continual finite element. Functional (1.16) is computed as the sum of functionals defined on discrete-continual finite elements

$$\Phi(\overline{U})=\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\Phi_{ij}(\overline{U}^{ij});\quad \Phi_{ij}(\overline{U}^{ij})=0.5\cdot(K^{ij}\overline{U}^{ij},\overline{U}^{ij})-(\overline{R}^{ij},\overline{U}^{ij}). \quad (3.4)$$

Stiffness matrix of element K^{ij} is computed with the use of method of basis variations

$$(K^{ij})_{st}=\widetilde{\Phi}_{ij}(\overline{e}_s+\overline{e}_t)-\widetilde{\Phi}_{ij}(\overline{e}_s)-\widetilde{\Phi}_{ij}(\overline{e}_t)+\widetilde{\Phi}_{ij}(\overline{e}_0),\ \ s,t=1,2,...,24, \quad (3.5)$$

where $(e_j)_k=\delta_{j,k}$; $(e_0)_k=0,\ k=1,2,...,24$; $\widetilde{\Phi}_{ij}(\overline{U}^{ij})=0.5\cdot(K^{ij}\overline{U}^{ij},\overline{U}^{ij})$;

$$K^{ij}=\begin{bmatrix} K_{uu}^{11} & K_{uv}^{11} & K_{uu}^{12} & K_{uv}^{12} & K_{uu}^{13} & K_{uv}^{13} & K_{uu}^{14} & K_{uv}^{14} \\ K_{vu}^{11} & K_{vv}^{11} & K_{vu}^{12} & K_{vv}^{12} & K_{vu}^{13} & K_{vv}^{13} & K_{vu}^{14} & K_{vv}^{14} \\ K_{uu}^{21} & K_{uv}^{21} & K_{uu}^{22} & K_{uv}^{22} & K_{uu}^{23} & K_{uv}^{23} & K_{uu}^{24} & K_{uv}^{24} \\ K_{vu}^{21} & K_{vv}^{21} & K_{vu}^{22} & K_{vv}^{22} & K_{vu}^{23} & K_{vv}^{23} & K_{vu}^{24} & K_{vv}^{24} \\ K_{uu}^{31} & K_{uv}^{31} & K_{uu}^{32} & K_{uv}^{32} & K_{uu}^{33} & K_{uv}^{33} & K_{uu}^{34} & K_{uv}^{34} \\ K_{vu}^{31} & K_{vv}^{31} & K_{vu}^{32} & K_{vv}^{32} & K_{vu}^{33} & K_{vv}^{33} & K_{vu}^{34} & K_{vv}^{34} \\ K_{uu}^{41} & K_{uv}^{41} & K_{uu}^{42} & K_{uv}^{42} & K_{uu}^{43} & K_{uv}^{43} & K_{uu}^{44} & K_{uv}^{44} \\ K_{vu}^{41} & K_{vv}^{41} & K_{vu}^{42} & K_{vv}^{42} & K_{vu}^{43} & K_{vv}^{43} & K_{vu}^{44} & K_{vv}^{44} \end{bmatrix};\ \ \overline{U}^{ij}=\overline{U}^{ij}(s)=\begin{bmatrix} \overline{U}_n^{11} \\ \overline{U}_n^{21} \\ \overline{U}_n^{12} \\ \overline{U}_n^{22} \end{bmatrix}. \quad (3.6)$$

Here $K_{uu}^{lm}, K_{uv}^{lm}, K_{vu}^{lm}, K_{vv}^{lm},\ l,m=1,2,3,4$ are matrices of the third order.

3.6. Construction of element matrices. Several auxiliary matrices are introduced:

$$K_{uu}^{ij}=\begin{bmatrix} K_{uu}^{11} & K_{uu}^{12} & K_{uu}^{13} & K_{uu}^{14} \\ K_{uu}^{21} & K_{uu}^{22} & K_{uu}^{23} & K_{uu}^{24} \\ K_{uu}^{31} & K_{uu}^{32} & K_{uu}^{33} & K_{uu}^{34} \\ K_{uu}^{41} & K_{uu}^{42} & K_{uu}^{43} & K_{uu}^{44} \end{bmatrix};\quad K_{uv}^{ij}=\begin{bmatrix} K_{uv}^{11} & K_{uv}^{12} & K_{uv}^{13} & K_{uv}^{14} \\ K_{uv}^{21} & K_{uv}^{22} & K_{uv}^{23} & K_{uv}^{24} \\ K_{uv}^{31} & K_{uv}^{32} & K_{uv}^{33} & K_{uv}^{34} \\ K_{uv}^{41} & K_{uv}^{42} & K_{uv}^{43} & K_{uv}^{44} \end{bmatrix}; \quad (3.7)$$

$$K_{vu}^{ij}=\begin{bmatrix} K_{vu}^{11} & K_{vu}^{12} & K_{vu}^{13} & K_{vu}^{14} \\ K_{vu}^{21} & K_{vu}^{22} & K_{vu}^{23} & K_{vu}^{24} \\ K_{vu}^{31} & K_{vu}^{32} & K_{vu}^{33} & K_{vu}^{34} \\ K_{vu}^{41} & K_{vu}^{42} & K_{vu}^{43} & K_{vu}^{44} \end{bmatrix};\quad K_{vv}^{ij}=\begin{bmatrix} K_{vv}^{11} & K_{vv}^{12} & K_{vv}^{13} & K_{vv}^{14} \\ K_{vv}^{21} & K_{vv}^{22} & K_{vv}^{23} & K_{vv}^{24} \\ K_{vv}^{31} & K_{vv}^{32} & K_{vv}^{33} & K_{vv}^{34} \\ K_{vv}^{41} & K_{vv}^{42} & K_{vv}^{43} & K_{vv}^{44} \end{bmatrix}. \quad (3.8)$$

3.7. Element local differential relations expresses stationarity condition of distinctive element free of other elements of finite element model and has the form

$$\begin{cases} \overline{v}^{ij}=\partial_s\overline{u}^{ij} \\ -K_{vv}^{ij}(\overline{v}^{ij})'+\widetilde{K}_{uv}^{ij}\overline{v}^{ij}+K_{uu}^{ij}\overline{u}^{ij}=\overline{R}_u^{ij} \end{cases} \quad (3.9)$$

where

$$\overline{R}_u^{ij}=\overline{R}_u^{ij}(s)=[(\overline{R}_{u,n}^{11})^T\ (\overline{R}_{u,n}^{21})^T\ (\overline{R}_{u,n}^{12})^T\ (\overline{R}_{u,n}^{22})^T]^T;\ \ \widetilde{K}_{uv}^{ij}=K_{uv}^{ij}-K_{vu}^{ij};\ \ K_{vu}^{ij}=(K_{uv}^{ij})^*; \quad (3.10)$$

$$\overline{u}^{ij}=\overline{u}^{ij}(s)=[(\overline{u}_n^{11})^T\ (\overline{u}_n^{21})^T\ (\overline{u}_n^{12})^T\ (\overline{u}_n^{22})^T]^T;\ \ \overline{v}^{ij}=\overline{v}^{ij}(s)=[(\overline{v}_n^{11})^T\ (\overline{v}_n^{21})^T\ (\overline{v}_n^{12})^T\ (\overline{v}_n^{22})^T]^T. \quad (3.11)$$

3.8. Assembling of global matrices. Global matrices K_{uu}, K_{uv}, K_{vu} and K_{vv} for the whole system of elements are assembled in accordance with conventional procedures of finite element method.

3.9. Assembling of global nodal load vector. The global nodal load vector has the form

$$\overline{R}=\overline{R}(s)=[(\overline{R}_u)^T\ \ 0]^T;\ \ \overline{R}_u=\overline{R}_u(s)=[\overline{R}_{u,n}^{11}\ \ \overline{R}_{u,n}^{12}\ \ ...\ \ \overline{R}_{u,n}^{N_1N_2}]^T. \quad (3.12)$$

3.10. Boundary conditions. Let $s_k,\ k=1,2,...,n_k$ be coordinates of boundary cross-sections of structure. Boundary conditions have the form

$$B_k^- \overline{U}_n(s_k-0)+B_k^+\overline{U}_n(s_k+0)=\overline{g}_k,\ k=1,2,...,n_k, \qquad (3.13)$$

where B_k^-, B_k^+ are matrices of boundary conditions of 6N order; $\overline{g}_k$ is the right-side vector of boundary conditions; $N = N_1 \cdot N_2$;

$$\overline{U}_n=\overline{U}_n(s)=[\,(\overline{u}_n)^T\ \ (\overline{v}_n)^T\,]^T; \qquad (3.14)$$

$$\overline{u}_n=\overline{u}_n(s)=[(\overline{u}_n^{11})^T\ (\overline{u}_n^{12})^T\ ...\ (\overline{u}_n^{N_1N_2})^T]^T;\quad \overline{v}_n=\overline{v}_n(s)=[(\overline{v}_n^{11})^T\ (\overline{v}_n^{12})^T\ ...\ (\overline{v}_n^{N_1N_2})^T]^T. \qquad (3.15)$$

3.11. Resolving multipoint boundary problem is the following system:

$$\left.\begin{aligned}&\overline{U}_n'(s)=A\overline{U}_n(s)+\overline{\overline{R}}(s)\\ &B_k^-\overline{U}_n(s_k-0)+B_k^+\overline{U}_n(s_k+0)=\overline{g}_k,\ k=1,2,...,n_k\end{aligned}\right\} \qquad (3.16)$$

where

$$A=\begin{bmatrix}0 & E\\ (K_{vv})^{-1}K_{uu} & (K_{vv})^{-1}\widetilde{K}_{uv}\end{bmatrix};\quad \overline{\overline{R}}=\overline{\overline{R}}(s)=-\begin{bmatrix}0\\ (K_{vv})^{-1}\overline{R}_u\end{bmatrix};\quad \overline{U}_n'=\partial_s\overline{U}_n; \qquad (3.17)$$

$$\widetilde{K}_{uv}=K_{uv}-K_{vu};\quad K_{vu}=K_{uv}^*. \qquad (3.18)$$

Strain and stress components are computed according to corresponding formulas.

4. ANALYTICAL SOLUTION OF MULTIPOINT BOUNDARY PROBLEMS IN STRUCTURAL ANALYSIS FOR SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS

4.1. Conventional formulation of multipoint boundary problem has the form

$$\overline{y}^{(1)}-A\overline{y}=\overline{f},\ \ x\in\bigcup_{k=1}^{n_k-1}(x_k,x_{k+1});\quad B_k^-\,\overline{y}(x_k-0)+B_k^+\,\overline{y}(x_k+0)=\overline{g}_k,\ \ k=1,...,n_k. \qquad (4.1)$$

where $\overline{y}=\overline{y}(x)=[\,y_1(x)\ \ y_2(x)\ \ ...\ \ y_n(x)\,]^T$ is the desirable vector function; $\overline{f}=\overline{f}(x)=[\,f_1(x)\ \ f_2(x)\ \ ...\ \ f_n(x)\,]^T$ is the right-side vector function; $x_k, r=1,...,n_k$ are coordinates of boundary points; A is the matrix of coefficients of order n; B_k^-, B_k^+ are matrices of boundary conditions of order n; $\overline{g}_k$ is the right-side vector of boundary conditions.

Solutions of multipoint boundary problems of this type in structural analysis are accentuated by numerous factors. They include boundary effects (stiff systems) and considerable number of differential equations (several thousands). Moreover, matrix of coefficients of a system normally has eigenvalues of opposite signs and its Jordan matrix is not diagonal. Special method of solution of multipoint boundary problems for systems of ordinary differential equations in structural analysis has been developed. Not only does it overcome all difficulties mentioned above but its major peculiarities also includes universality, computer-oriented algorithm, computational stability, optimal conditionality of resultant systems and partial Jordan decomposition of matrix of coefficient, eliminating necessity of calculation of root vectors.

4.2. Formulation of multipoint boundary problem with the use of distributions has the form

$$\overline{Y}'=A\overline{Y}+\overline{F},\ x\in(-\infty;+\infty);\quad B_k^-\,\overline{Y}(x_k-0)+B_k^+\,\overline{Y}(x_k+0)=\overline{g}_k,\ \ k=1,...,n_k, \qquad (4.2)$$

where

$$\overline{F}=\overline{f}+\sum_{k=1}^{n_k}\delta(x-x_k)\Delta\,\overline{y}_k;\quad \Delta\,\overline{y}_k=\overline{y}(x_k+0)-\overline{y}(x_k-0); \qquad (4.3)$$

Y – generalized desirable vector function.

4.3. Jordan decomposition of matrix of coefficients has the form

$$A=T\,J\,T^{-1},\ \text{ where }\ J=\{J_1,J_2,...,J_u\}; \qquad (4.4)$$

T is the matrix of order n, which columns are eigenvectors and root vectors of matrix A; J is Jordan matrix of order n; J_p is Jordan cell corresponding to eigenvalue λ_p; $\dim J_p = m_p$.

At the present time there are no effective numerical methods of calculation of Jordan decomposition in the general case. Meanwhile the number of multiple eigenvalues in the considering type of problems normally limited. Besides they are generally zeros. In this connection special alternative approaches of solution have been developed.

4.4. Partial Jordan decomposition is based on computation of right and left eigenvectors

$$A = A_1 + A_2; \quad A_1 = T_1 J_1 \tilde{T}_1; \quad A_2 = A - A_2; \tag{4.5}$$

T_1 is the matrix containing right eigenvectors corresponding to non-zero eigenvalues of matrix A; $\tilde{T}_1$ is the matrix containing left eigenvectors corresponding to non-zero eigenvalues of matrix A; J_1 is diagonal Jordan matrix corresponding to non-zero eigenvalues; A_2 is the part of matrix A corresponding to prime and multiple zero eigenvalues.

4.5. Construction of projectors. Eigenvalues are renumbered according to the condition

$$\left.\begin{array}{l} \forall\lambda_p,\ \ p = 1,\dots,l \ \ \exists m_p = 1 \\ \forall\lambda_p,\ \ p = l+1,\dots,u \ \ \exists m_p > 1 \end{array}\right\}; \quad l = \sum_{p=1}^{u} \delta_{1,m_p} \tag{4.6}$$

Due to distinctive procedure, we should properly modify matrices T_1, $\tilde{T}_1$ and J_1.

Let P_1 and P_2 be projectors to subspaces of left and right eigenvectors and root vectors corresponding to non-zero and zero eigenvalues. They may be denoted as

$$P_1 = T_1(\tilde{T}_1 T_1)^{-1}\tilde{T}_1; \quad P_2 = E - P_1. \tag{4.7}$$

4.6. Construction of fundamental matrix-function of system of equations. Fundamental matrix-function is constructed in the special form convenient for problems of structural mechanics

$$\varepsilon(x) = T_1 \tilde{\varepsilon}_0(x)\tilde{T}_1 + \chi(x,0)[P_2 + \sum_{k=1}^{m_{max}-1} \frac{x^k}{k!} A_2^k], \tag{4.8}$$

where $m_{max} = \max_{l \le i \le u} m_i$;

$$\chi(x,\lambda_p) = \begin{cases} \mathrm{sign}(x)\theta(-\mathrm{Re}(\lambda_p)x), & \lambda_p \ne 0 \\ 0.5\,\mathrm{sign}(x), & \lambda_p = 0 \end{cases}; \quad \tilde{\varepsilon}_0(x) = \mathrm{diag}\{\chi(x,\lambda_1)\exp(\lambda_1 x),\dots,\chi(x,\lambda_l)\exp(\lambda_l x)\}. \tag{4.9}$$

It should be stated that the sum in the right side of (4.8) contains four or lower components and corresponds to so-called beam part of solution of system.

4.7. General solution of problem. General solution of problem (4.2) is defined by formula

$$\overline{Y} = \varepsilon * \overline{F} = \varepsilon * \overline{f} + \sum_{k=1}^{n_k} \varepsilon(x - x_k)\overline{C}_k, \tag{4.10}$$

where $\overline{C}_k = \Delta\overline{y}_k$, $k = 1,\dots,n_k$ are vectors of coefficients; * is the symbol of convolution.

4.8. Computation of constant coefficients in general solution from boundary conditions. Both explicit matrix method and method of basis variations can be used for computation of constant coefficients in general solution. Key features of the latter are algorithmic demonstrativeness and simplicity of computer realization.

5. COMPUTER REALIZATION OF DISCRETE-CONTINUAL FINITE ELEMENT METHOD OF STRUCTURAL ANALYSIS AND SOFTWARE

Discrete-continual finite element method has been realized in finite element analysis system SAFEM3D. The main purpose of this software is structural analysis with the use of discrete-continual finite element method. Programming environment is Compaq Visual Fortran 6.5 Professional. Program is designed for Microsoft Windows 95/98/NT/2000/ME/XP.