Application of Qualitative Methods to Research of Polyharmonic Oscillations

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1. Introduction

The development of the qualitative methods of investigation of dynamic systems, suggested by the authors, is the effective means for identification of dynamic systems. The results of the extensive investigations of the behaviour of linear dynamic systems and symmetrical system with double well potential under polyharmonic excitation are given in the paper. The bases of the method of qualitative investigation of oscillations were developed by Poincare. Application of these methods is most effective for the investigation of oscillations of systems with one degree of freedom. The classical approach to qualitative investigation of oscillations consists in finding out special points on a phase plane $(y, \dot{y})$ and definition of their type (node, saddle, centre or focus). Studying of special points of system explains the picture of trajectories of points on a phase plane (displacement, velocity) in their neighbourhood, however does not allow to study oscillatory processes finally.

Phase space of dynamic systems is multi-dimensional. Each point of this space is characterized by not less than four co-ordinates. In particular: displacement, velocity, acceleration and time. Real space has three dimensions. It is more convenient for the analysis. We consider the phase space as limited to three dimensions, namely displacement, velocity and acceleration. Another choice of parameters of phase planes is also possible [1, 2]. Phase trajectory on a plane $(y, \ddot{y})$ is of the greatest interest. It is known that accelerations of points are more sensitive to deviations of oscillations from harmonic ones.

It is connected with the fact that power criteria on it are interpreted most evidently. Besides, dependence $\ddot{y}(y)$ is back symmetric relative to axis y of the diagram of elastic characteristic. For example, in Figure 1 diagrams of change of the elastic characteristic and acceleration for the system with "backlash" are shown. Only the phase trajectories $\ddot{y}(y)$ allow establishing a type and a level of non-linearity of a system.

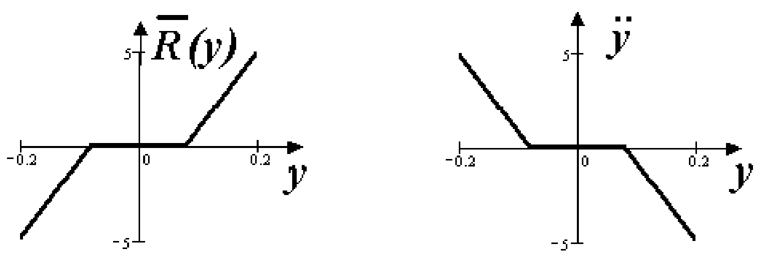

Figure 1. Diagrams of change of elastic properties and acceleration for system with "backlash".

The results of the extensive investigations of the dynamic systems behaviour under polyharmonic excitation are given in the paper.

2. Differential equation of polyharmonic forced oscillations

Let's remark that outer excitation can contain some harmonics for wide range of mechanical dynamic systems. Their amplitudes might be various. The forced oscillations of such systems described by the non-linear differential equation of the type

$$\ddot{y} + \varepsilon \dot{y} + R(y) = F(t);$$

$$F(t) = F_0 + \sum_{i=1}^{n} F_i(t)\cos(\omega_i t) + \sum_{j=1}^{n} F_j(t)\sin(\omega_j t), \quad (1)$$

$$i = 1, 2, 3 \ldots n, \; j = 1, 2, 3 \ldots n,$$

where y is the generalized coordinate; ε is the damping coefficient of the system; $R(y)$ is the elastic characteristic of the system; and F_0, F_i, F_j, ω_i are parameters of the outer polyharmonic excitation.

Let's restrict our investigation to symmetrical biharmonic oscillations, then outer polyharmonic excitation has the form:

$$F(t) = F_1 \cos(\omega_1 t) + F_m \cos(\omega_m t), \; m = 1,2,3\ldots, \quad (2)$$

The excitation is monoharmonic in a case if $m = 1$. The results of investigation for $m = 2, 3$ are presented in the paper. We compare linear system to nonlinear symmetric system with double well potential (buckling).

3. The methods of modelling

The hybrid computing complexes (HCC) present the synthesis of analogue and numerical computers. They possess the fastness of the analogue and the precision of the numerical computers with large memory size. The HCC gives the possibility to observe visually the computing process during the investigations by means of oscillographs, self-recorders etc. Besides, it is possible to change the parameters of the investigated system in the process of computing.

The investigation of the forced oscillation systems with buckling was carried out on the HCC produced on the base of IBM PC and analogue computer ACC-31 with the signal generator of special shape. The maximum output signal constitutes 10 V within the frequency range 0.001-10 kHz. The double-trace oscillograph C1-99 was used for visual observation of the computing process - electric signals from the major amplifier outputs. The results of the non-linear differential equation system integration were transmitted by means of the interface devices to IBM PC.

The standard mathematical securing is used for the analogue-to-digital converter functioning. The information input into IBM PC is stored on the hard disk in the text file form. The spectral characteristics of the oscillating processes are obtained by means of the standard programme of the fast Fourier transformation. The standard graphic software package is used for the graphic presentation of the dynamic processes.

4. Analysis of biharmonic oscillations of the linear systems

The system with a linear elastic characteristic has been adopted as a reference. The elastic characteristic in this case has the following form:

$$R(y) = \alpha y, \quad (3)$$

The values of system parameters have been taken as follows: α=408 s^{-2}; $\varepsilon = 0.1;0.5;1\ s^{-1}$; $F_1 = 0.5;1;1.5\ ms^{-2}$, $F_{2,3} = 0.05\ldots4.5\ ms^{-2}$.

The general forms of amplitude-frequency characteristics of system (1) are given in Figure 2.

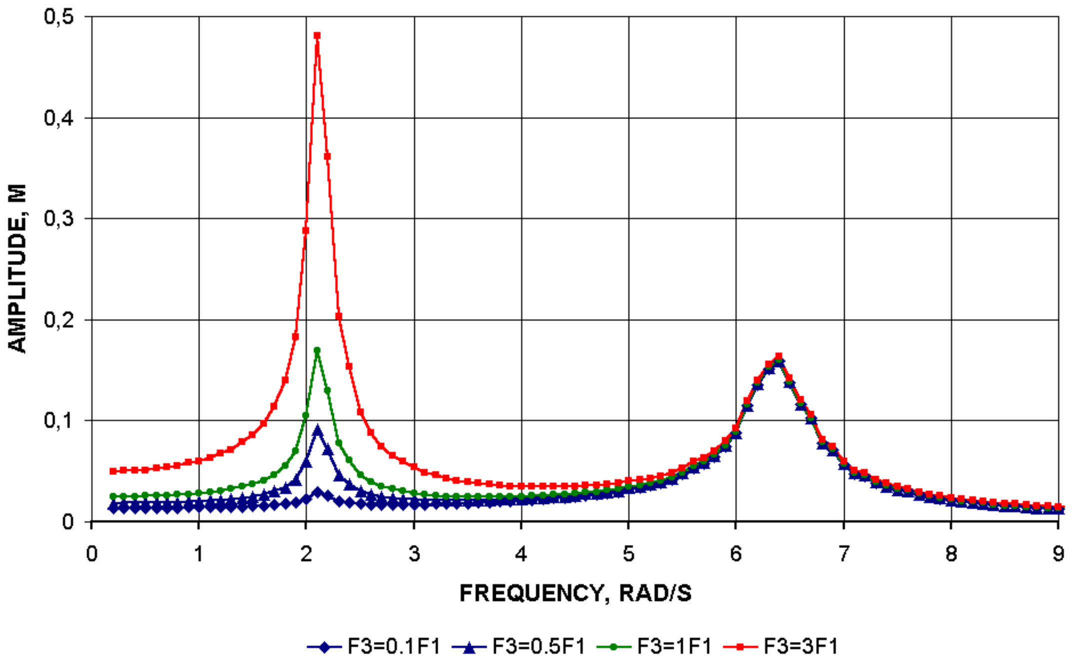

Figure 2. Amplitude-frequency characteristics of the linear system: $m = 3$; $\varepsilon = 0.5\ s^{-1}$; α=408 s^{-2}; $F_1 = 0.5\ ms^{-2}$.

As it is shown in Figure 2, the linear dynamic system with one degree of freedom can have an infinite number of resonance zones on harmonics with multiple frequencies according to conditions $\omega_\mu = \mu\omega_1 = \omega_0$, μ=0,1,2,3,... .

The stable branches of amplitude-frequency characteristics form two frequency ranges with considerably different behaviour. As it is seen from the results presented in Figure 2, range I is the area of appearance of combinative oscillations, range II is the area of resonance oscillations of the fundamental tone.

The time processes, phase trajectories and spectral densities of the forced oscillation energy distribution at different amplitudes of the outer excitation are shown in Figure 3.

Within the first frequency range beat-like oscillations arise. The amplitudes of fundamental harmonic and sub-harmonic are commensurable. The position of oscillations centre is not constant. That is why additional closed loops appear on the Poincare map. The additional closed loops occur as well on phase trajectories on a plane $(y, \ddot{y})$. It is necessary to note that all of them are parallel. The angle of lean of these pathways to an axis y is $-\omega^2$.

The amplitude of sub-harmonic oscillations is too small in a second resonance zone. The time processes $y(t)$, $\dot{y}(t)$ and $\ddot{y}(t)$ look like monoharmonic. Meanwhile, the phase trajectories $(y, \ddot{y})$ and $(\dot{y}, \ddot{y})$ have some peculiarities. The phase trajectories on a plane $(y, \ddot{y})$ have two additional loops on their ends. They are symmetrical to a "skeleton" curve. The phase trajectories on a plane $(\dot{y}, \ddot{y})$ are ellipses. Their main axes are inclined. The angle of lean of axes depends on a ratio of phases of the outer excitation.

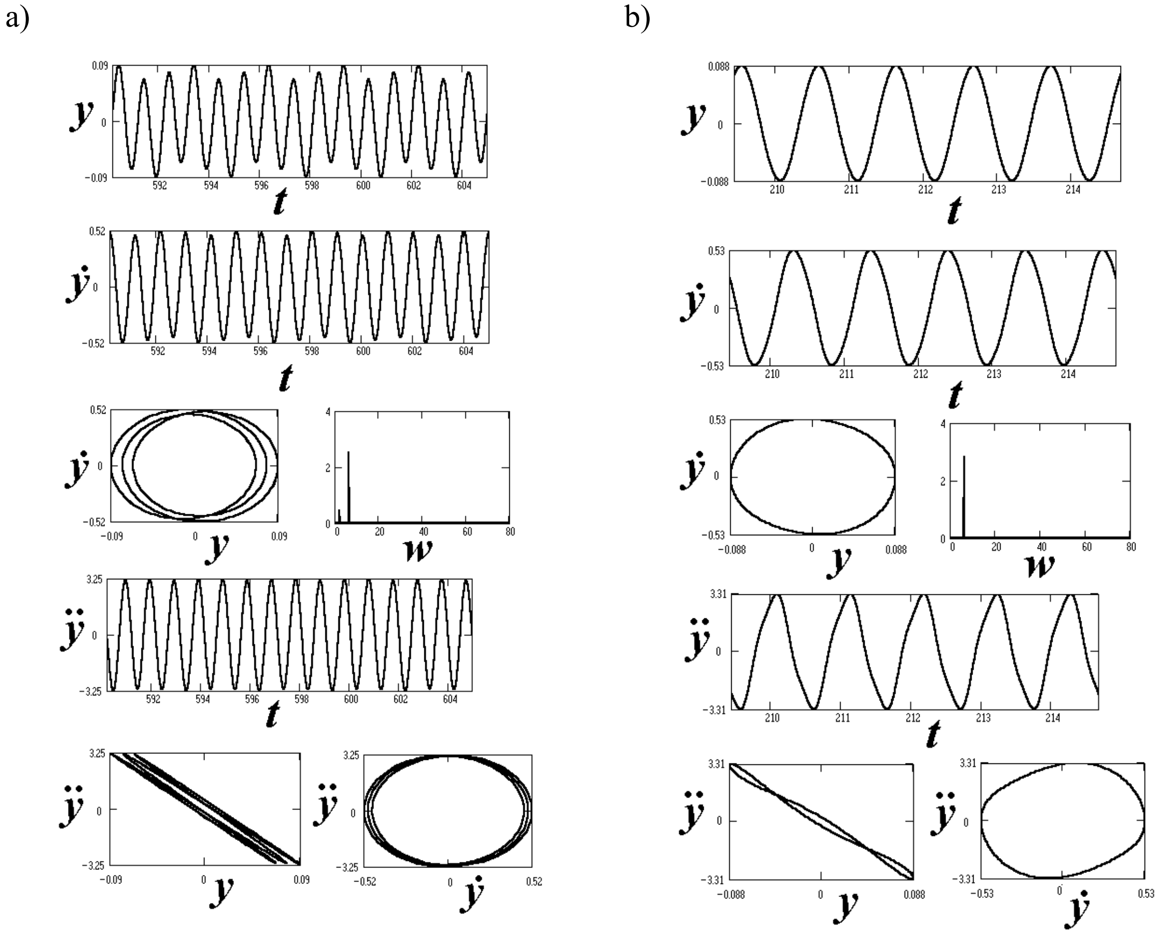

Figure 3. Time characteristics and phase trajectories: $m = 3\,;\varepsilon = 0.1\ s^{-1};\ F_1 = 0.5\ ms^{-2}\ F_3 = 0.25\ ms^{-2}$: a) resonance oscillation at the frequency $\omega = 2.19\ rad/s$.; b) resonance oscillation at the frequency $\omega = 6.39\ rad/s$.

5. Analysis of biharmonic oscillations of the systems with double well potential

The dynamic behaviour of such systems is described by the non-linear differential Duffing-type equation. The elastic characteristic has the following form:

$$R(y) = -\alpha\, y + \beta y^3, \qquad (4)$$

The values of system parameters have been taken as follows: $m = 2;3\,;\varepsilon = 0.1;0.5;1\ s^{-1};\ \alpha = 408\ s^{-2};\ \beta = 7660000\,m^{-2}s^{-2}$; $F_1 = 0.15;\,0.5;\,1.5\ ms^{-2};\ F_1 = 0.015\ldots3\ ms^{-2}$.

The existence of one from three stable oscillation regimes is possible depending on the potential energy value in the system:

- “large” oscillations relative to all three equilibrium point;
- “small” oscillations relative to the upper equilibrium point;
- “small” oscillations relative to the lower equilibrium point.

The “large” oscillations possess the peculiarities of the rigid system behaviour, and “small” oscillations possess the qualities of soft systems. The character of the oscillation amplitude changing with the increase or decrease of the excitation frequencies is presented in Figure 4. The stable branches of the amplitude-frequency characteristic form five frequency ranges, for which the graphic of the time process, phase trajectories on planes and spectral characteristics are obtained. The stalls of the forced oscillation regimes from one branch to another are accompanied not only by the transition from "large" oscillations to "small", or vice versa, but also by the appearance of the combination tones.

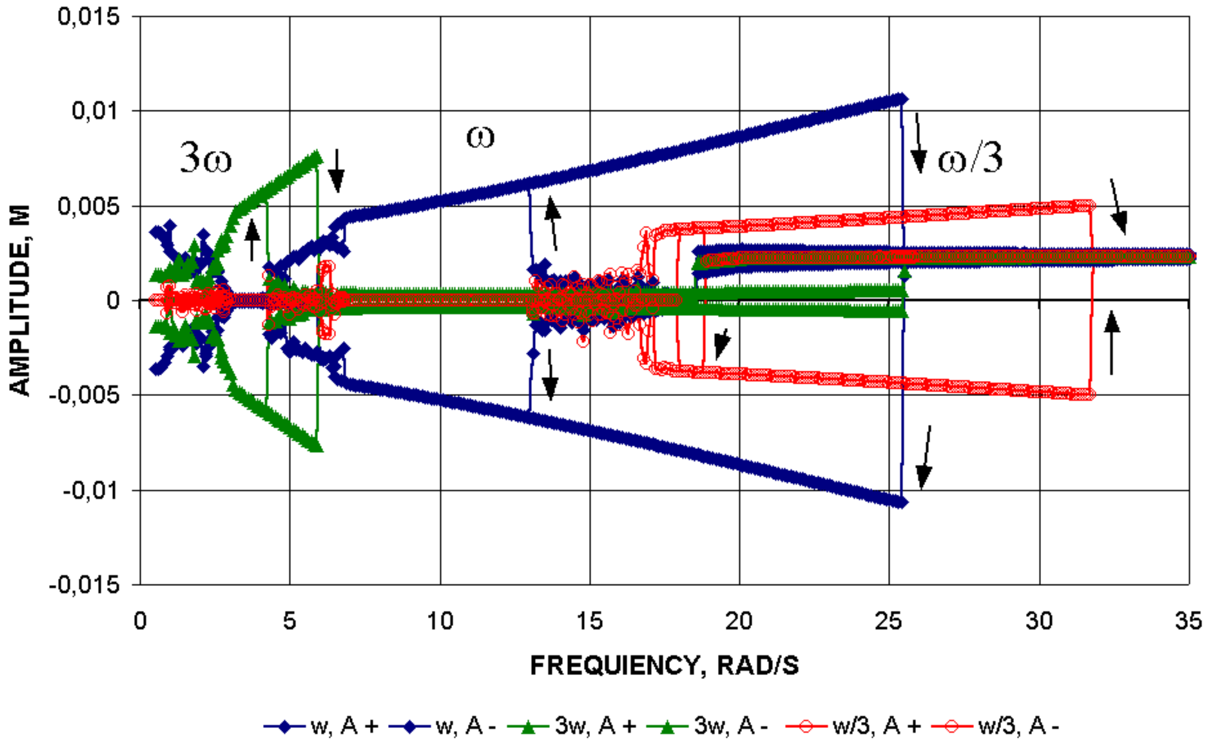

Figure 4. Amplitude-frequency characteristics of the system with double well potential: $m = 3$; $\varepsilon = 0.5\ s^{-1}$; $\alpha = 408\ s^{-2}$; $\beta = 7660000\, m^{-2} s^{-2}$; $F_1 = 0.15\ ms^{-2}$; $F_1 = 0.075\ ms^{-2}$.

Range I $(\omega = 0 \div 3 rad/s)$ is the area of the laying-on of ultra-harmonic "small" oscillations of $n\omega$ $(n = 2,3,4,5\ldots)$ order on the "large" oscillations of the fundamental tone both at increasing and decreasing of the excitation frequencies.

Range II $(\omega = 3 \div 7 rad/s)$ is the area of the "large" ultra-harmonic oscillations of $3\,\omega$ order at the excitation frequency.

Range III $(\omega = 7 \div 26 rad/s)$ is the area of the "large" oscillations of fundamental tone at the increase of excitation frequencies and the combination with the "small" ultra- and subharmonic oscillations of 2ω, 3ω and $\omega/2$ order at the excitation frequency decrease. It should be noted that oscillations on even harmonics are not stable because the system is symmetrical. The appearance of chaotic oscillations is also observed within this range.

Range IV is the area of "large" sub-harmonic oscillations of $\omega/3$ order both at increasing and decreasing the excitation frequencies.

Range V is the super-resonance area where only "small" oscillations exist. In this area the forced oscillations are possible relative to one equilibrium condition as well as to another non-adjacent to it.

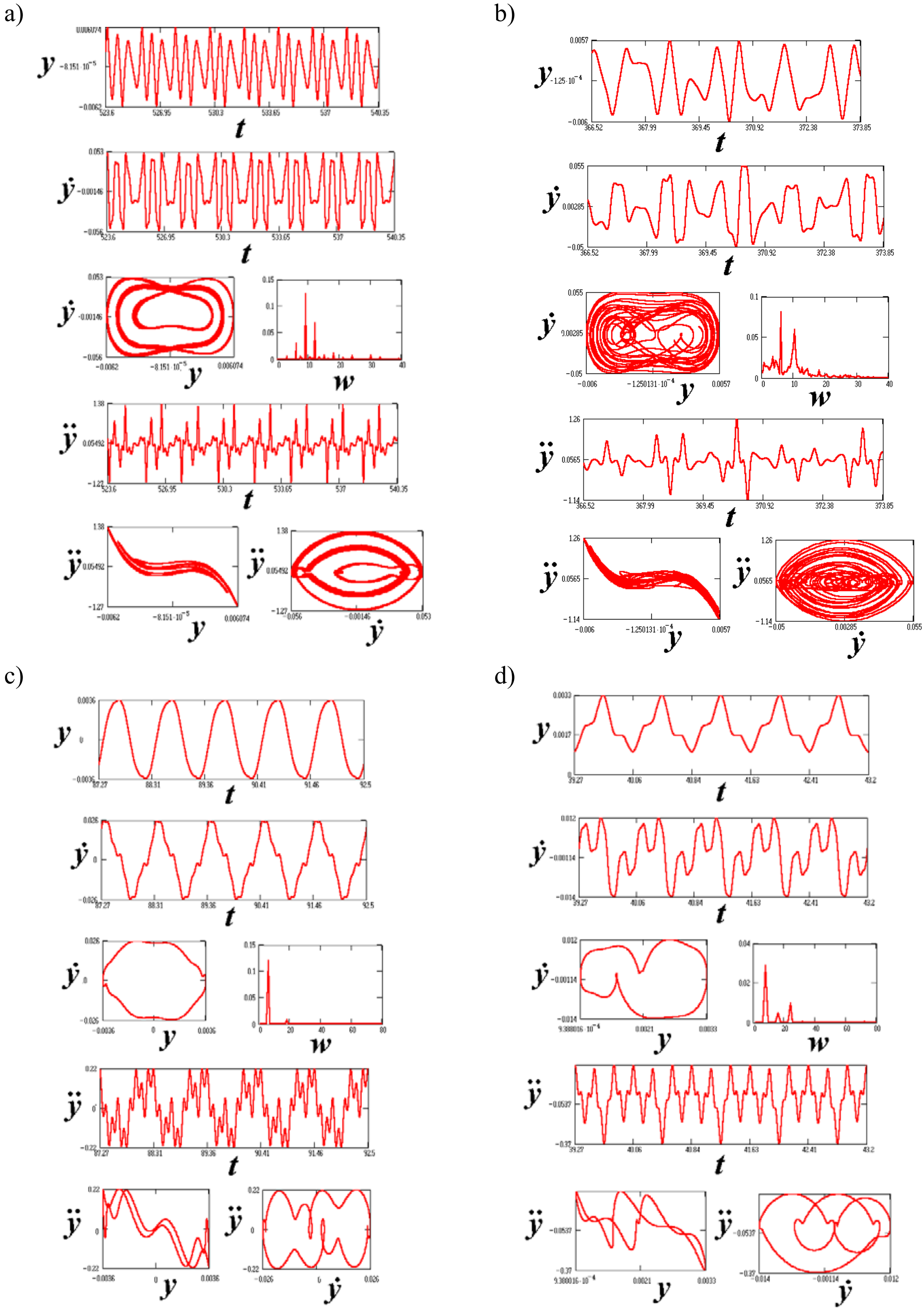

Figure 5. Time characteristics and phase trajectories of the system with double well potential ($m = 3$; $\varepsilon = 0.5\ s^{-1}$; $\alpha = 408\ s^{-2}$; $\beta = 7660000\, m^{-2} s^{-2}$; $F_1 = 0.15\ ms^{-2}$; $F_1 = 0.075\ ms^{-2}$): a) combinative oscillations; b) chaotic oscillations; c) “large” sub-harmonic oscillations; d) “small” oscillations.

The possibility of occurrence of the non-adjacent stable oscillations at the fixed frequency of excitation is the peculiarity of the investigated systems. The realization of one of the stable regimes of oscillations depends on the initial conditions in a complicated manner.

The frequencies of "large" oscillations stall for the cases of monoharmonic and biharmnoic excitation are different. It is important that "skeleton" curves for oscillations on fundamental tone, ultra- and sub-harmonic oscillations have different angles. The amplitude of oscillations within the frequency range III is larger then if it was a monoharmonic excitation.

As shown in Figure 5 a-c, for all the types of "large" oscillations the phase trajectories are back symmetrical relative to axis y of the diagram of elastic characteristic. It allows to recognise the type of dynamic system.

The development of qualitative methods of investigation of dynamic systems suggested by the authors is effective means of analysis and identification of dynamic systems. Simultaneous use of all three types of signals registered in time, namely displacement, velocity and acceleration allows to expand considerably the opportunities of traditional methods of investigation. The use of the given phase trajectories enables us to determine with a high degree of reliability the following peculiarities:

- presence or absence of non-linear character of behaviour of a dynamic system;
- type of non-linearity;
- type of dynamic process (oscillations of the basic tone, combinative oscillations, chaotic oscillations.).

Unlike existing asymptotic and stochastic methods of identification of dynamic systems, the use of the suggested technique is not connected with the use of a significant amount of computing procedures, and also it has a number of advantages at the investigation of complicated oscillations.

References

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