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AN APPLICATION OF FORMAL POWER SERIES FOR THE DEVELOPMENT OF OPTICAL FILTERS

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Abstract. *The application of a recent method using formal power series is proposed. It is based on a new representation for solutions of Sturm-Liouville equations. This method is used to calculate the transmittance and reflectance coefficients of finite inhomogeneous layers with high accuracy and efficiency. Tailoring the refraction index profile defining the inhomogeneous media it is possible to develop very important applications such as optical filters. A number of profiles were evaluated and then some of them selected in order to perform an improvement of their characteristics via the modification of their profiles.*

1 INTRODUCTION

Numerous applications where the problem of finding the reflectance and transmittance coefficients of an inhomogeneous layer arises are found in optical physics, thin-film optics, and modern engineering, among other fields. The problem has been studied using numerous methods (see, e.g., [1], [2], [3], and the references therein) like the WKB, the Plane-Wave Expansion (PWE), and the Transfer-Matrix Method (TMM).

Among the different numerical methods that exist nowadays to solve this problem we use one that was recently developed (see [4]) and which is based on a new representation for solutions of Sturm-Liouville equations obtained in [5] and developed at [6]. The solution there is represented in the form of a functional series. The method there proposed allows an accurate and efficient calculation of the reflectance and transmittance coefficients.

In particular we concentrate here in applications related to the filtering of optical signals which is of great importance in communication systems engineering. In this field it is possible to use the method in order to characterize optical filters based on inhomogeneous profiles with sinusoidal and chirped behaviors.

2 TRANSMITTANCE AND REFLECTANCE CALCULATION

According to [4], the inhomogeneous media under consideration will have the structure shown in Figure 1.

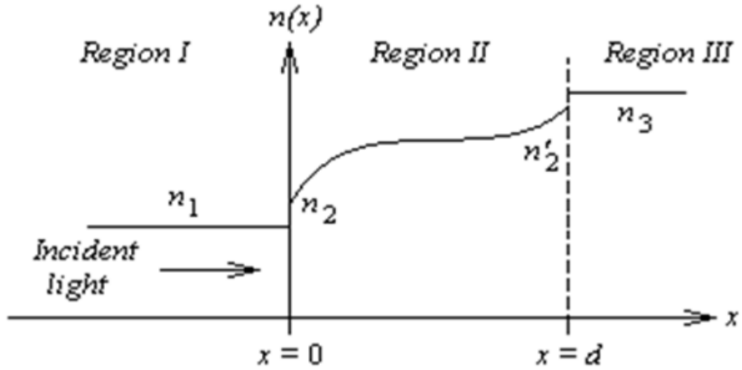

Figure 1. An inhomogeneous layer or "index profile".

The refractive index n has constant values n_1 and n_3 in the regions I and III respectively and is an arbitrary continuous function in the region II. If we suppose an incident wave in region I (with normal incidence, for the sake of simplicity) represented by the scalar function u which stands for a transverse component of the electric field of an s-polarized electromagnetic wave, the following Helmholtz equation is satisfied

$$u''(x) + [k^2 n^2(x)]u(x) = 0 \tag{1}$$

where k is the free-space circular wavenumber. If the incident wave is supposed to have the form $e^{-ik_1 x}$, where $k_1 = kn_1$, then together with the reflected wave the whole solution for $x < 0$ is

$$u(x) = e^{-ik_1 x} + Re^{ik_1 x}; \quad x < 0$$

where the constant R is the reflection coefficient whose absolute value is less than 1. The solution corresponding to the transmitted wave in region III has the form

$$u(x) = Te^{-ik_3x}, \quad x > d$$

where T is the transmission coefficient and $k_3 = kn_3$. In this case and for non absorbent media the following energy conservation relation holds

$$|R|^2 + n_3|T|^2/n_1 = 1. \tag{2}$$

The general solution of (1) for $0 < x < d$ was proposed to have the form

$$u(x) = c_1u_1 + c_2u_2$$

and consists of two linearly independent solutions u_1 and u_2 in the interval $0 \le x \le d$ such that

$$u_1(0) = 1, \qquad u_1'(0) = 0, \tag{3}$$

$$u_2(0) = 0, \qquad u_2'(0) = 1, \tag{4}$$

and with c_1 and c_2 being arbitrary constants. So, from the continuity and initial conditions the expressions for R and T were found to be

$$R = \frac{-k_1k_3u_2(d) - u_1'(d) - ik_3u_1(d) + ik_1u_2'(d)}{[u_1'(d) - k_1k_3u_2(d)] + i[k_3u_1(d) + ik_1u_2'(d)]},$$

$$T = \frac{2ik_1[u_1(d)u_2'(d) - u_1'(d)u_2(d)e^{ik_3d}]}{[u_1'(d) - k_1k_3u_2(d)] + i[k_3u_1(d) + k_1u_2'(d)]}.$$

Solutions u_1 and u_2 are calculated as

$$u_1(x) = \sum_{m=0}^{\infty} \frac{\tilde{X}^{2m}(x)}{2m!}, \qquad u_2(x) = \sum_{m=0}^{\infty} \frac{X^{(2m+1)}(x)}{(2m+1)!},$$

where $X^{(0)}(x) \equiv \tilde{X}^{(0)}(x) \equiv 1$, and for $m \in \mathbf{N}$

$$X^{(m)}(x) = m\int_0^x X^{(m-1)}(\xi)d\xi \qquad \text{for odd } m$$

$$X^{(m)}(x) = m\int_0^x X^{(m-1)}(\xi)q(\xi)d\xi \qquad \text{for even } m$$

$$\tilde{X}^{(m)}(x) = m\int_0^x \tilde{X}^{(m-1)}(\xi)q(\xi)d\xi \qquad \text{for odd } m$$

$$\tilde{X}^{(m)}(x) = m\int_0^x \tilde{X}^{(m-1)}(\xi)d\xi \qquad \text{for even } m$$

and where $q \in C[0, d]$ and u are complex valued functions of an independent real variable x in $[0, d]$, and $q(x) = -k^2n^2(x)$.

3 OPTICAL FILTERING

Let us consider an inhomogeneous layer whose index profile is given by

$$n(x) = n_2 + n_4 \cos(Kx)$$

where n_2, n_4, and K are constants. The constant n_2 is the averaged index of refraction of the medium, n_4 may be regarded as the depth of the sinusoidal index modulation, and K is related to the period of the index variation Λ by

$$K = 2\pi / \Lambda.$$

The depth of modulation is often much smaller than the averaged index of refraction (i. e., $n_4 << n_2$). The constant K is also called the *grating momentum* or the *grating wave vector*. Spectral filters based on the sinusoidal variation of the refractive index are known as Bragg reflectors.

When specially tailored, Bragg reflectors allow to obtain reflectance responses like the one shown in Figure 2 (see [7]), where it is possible to see that beyond a band where reflectance almost reach the 100% it rapidly decays on both sides. In Figure 2, $\Delta k = 2k_0 - K$ is a momentum mismatch, κ is a coupling constant $\kappa = \pi n_4 / \lambda$, and $L = d$, the width of the inhomogeneous layer.

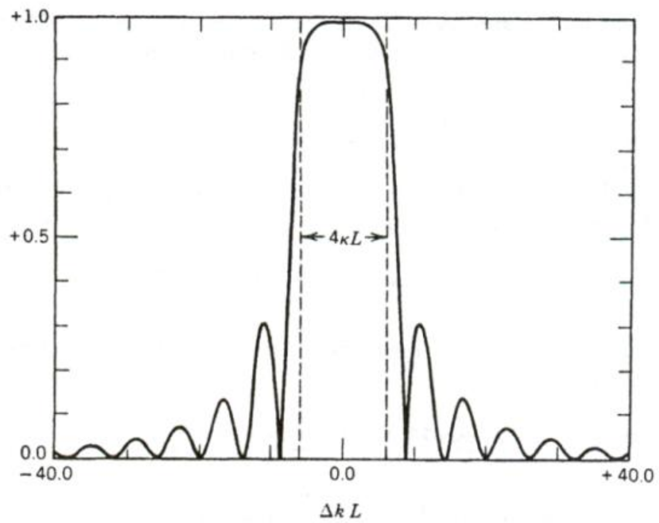

Figure 2. The reflectance of a Bragg reflector calculated by using the coupled-mode theory ($|\kappa|L = 3.0$) [7].

Filters carry out an extremely important function in WDM/DWDM (Wavelength Division Multiplexing/Dense WDM) equipment on the demultiplex side separating the received channels and have recommended parameter values such as: maximum insertion loss of 1.5 dB, passband; minimum insertion loss of 40 dB, stopband; and optical reflectance of –40 dB [8].

4 EVALUATION OF SOME PROFILES

With the aid of Matlab it is possible to calculate numerically the reflectance and transmittance intensities of a number of profiles with sinusoidal behaviors and to obtain curves like the ones shown in Figure 3, which in turn can be used to characterize the proposed filters. Some of them were selected in order to alter their characteristics and improve their behavior.

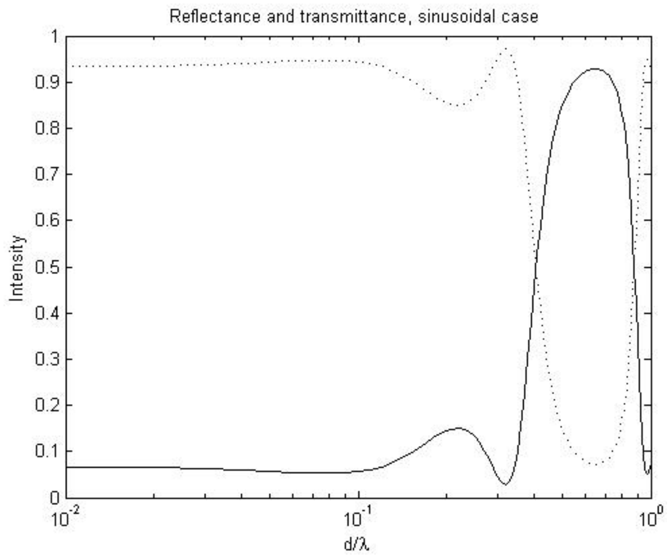

Figure 3. Reflectance (solid curve) and transmittance (dotted curve) intensities for a sinusoidal profile with $n_1 = 1$, $n_3 = 1.7$, $n(x) = 1.54 + \sin(4\pi x)$.

The main elements to consider are precision (which can be prescribed), center wavelength of the filter (since Figure 3 is in terms of d/λ one of both parameters –in general d– should be fixed in order to consider a specific range of wavelengths), bandwidth, minimum insertion loss, and Full Width at Half Maximum (FWHM). In the particular case of Figure 3 this parameters are: precision of 1×10^{-8}; bandwidth of 100 nm; maximum insertion loss: near 0.4 dB (but it should be remembered that a non absorbent medium was considered so this loss is only due to the inhomogeneous layer reflection, and its value should be reduced); minimum insertion loss: near 8 dB –its value should be increased–; and optical reflectance of near –13 dB, value which should also decrease.

A change like an increase of the periods in the layer leads to changes like the ones seen in Figure 4.

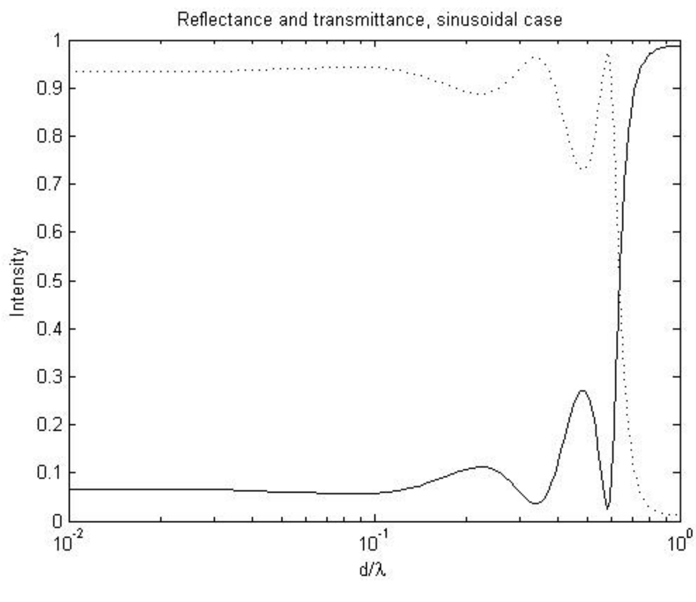

Figure 4. Reflectance (solid curve) and transmittance (dotted curve) intensities for a sinusoidal profile with $n_1 = 1$, $n_3 = 1.7$, $n(x) = 1.54 + \sin(6\pi x)$.

It is possible to see that for the same precision and bandwidth the maximum insertion loss decreased but the minimum insertion loss also decreased, while the reflectance almost kept unchanged. Another change is appreciated in the center wavelength, specified by d/λ.

It is important to note that the profiles for $n(x)$ in Figures 3 and 4 violate the condition that $n(x) \geq 1$, which implies the speed of light in some of the regions of the inhomogeneous media should be bigger than the speed of light in vacuum, which is not possible. For that reason, those cases should only be considered as good starting points for the process of optimization of the filter characteristics that we are trying to obtain. The final approximation to a filter, avoids the values of $n(x) \leq 1$.

Some other changes like different values of n_1, n_2, n_3 and n_4 were tested, as well as different number of periods of the non homogeneity and even phase changes in the sinusoidal profile. Precision was also diminished but only to reduce the time of calculation.

In Figure 5, a number of graphics showing the process of optimization are presented. It is possible to see how the main lobe starts to get narrower and tends to the desired level of 1. However, undesired effects also appear in the form of a growing side lobe and of a second lobe. Figure 5 a) corresponds to the best Maximum Insertion Loss obtained (0.06 dB) but it is a case of $n(x) \leq 1$ and shows to be not very selective with the wavelengths. Its values are $n_1 = 1$, $n_3 = 1.7$, $n(x) = 1.54 + \sin(6\pi x)$ (the same as in Figure 4). Figure 5 b) corresponds to the values $n_1 = 1$, $n_3 = 1.7$, $n(x) = 1.54 + 0.5\sin(6\pi x)$ where the condition $n(x) \geq 1$ is now fulfilled, but anyway it is almost impossible to get materials with $n \approx 1$ that can change their index continuously. Its main lobe is narrower and the side lobe seems to be smaller, but there is also a reduction in the intensity for the main lobe. Figure 5 c) uses an increase in the refraction index parameters in order to the filter to be more practical for construction. The effects are the same that in the previous case, namely a reduction in the bandwidth, and in the main and side lobe intensities. Its values are $n_1 = 1.3$, $n_3 = 2.0$, $n(x) = 1.84 + 0.5\sin(6\pi x)$. Figure 5 d) uses the same values but increases the amplitude of the sine to 0.8 which results in a rise in the amplitude of the intensity of the reflection. Figures 5 e) - 5 j) correspond to new increases in the values of refraction index and amplitude of the sine and their respective values are: for e) $n_1 = 1.7$, $n_3 = 2.4$, $n(x) = 2.24 + 0.8\sin(6\pi x)$; for f) $n_1 = 2.1$, $n_3 = 2.8$, $n(x) = 2.64 + 0.9\sin(6\pi x)$; for g) $n_1 = 2.1$, $n_3 = 2.8$, $n(x) = 2.64 + 1.1\sin(6\pi x)$; for h) $n_1 = 2.4$, $n_3 = 3.1$, $n(x) = 2.94 + 1.3\sin(6\pi x)$; for i) $n_1 = 2.4$, $n_3 = 3.1$, $n(x) = 2.94 + 1.6\sin(6\pi x)$; and for j) $n_1 = 2.9$, $n_3 = 3.5$, $n(x) = 3.44 + 2.0\sin(6\pi x)$.

The resulting characteristics for the case of filter j) are the following: Maximum Insertion Loss of 0.1 dB, Minimum Insertion Loss: 6.2 dB (even when it is difficult to identify a stop band), reflectance of –15.8 dB. If operation is considered for the $\lambda = 1530$ nm band the width of the inhomogeneous layer is 673.2 nm, FWHM is of more than 1000 nm which is too much if we consider that we want to filter a single signal with a bandwidth of less than 1 nm, so the resulting filter should be improved or used for wideband applications. If we also remember the values at the end of Section 3, it is possible to see that a number of values do not satisfy the requirements of a good filter. However, we think that the possibility to keep on looking for the right parameters should bring more light to this area.

Finally, the program performed the calculations in times from a couple of seconds to 2.7 hours, depending on the range considered, the requested precision and on the parameters of the amplitude of the sine, the refraction index, and the d/λ ratio. The number of powers used to approximate the solution were in the range of 5 to 211.

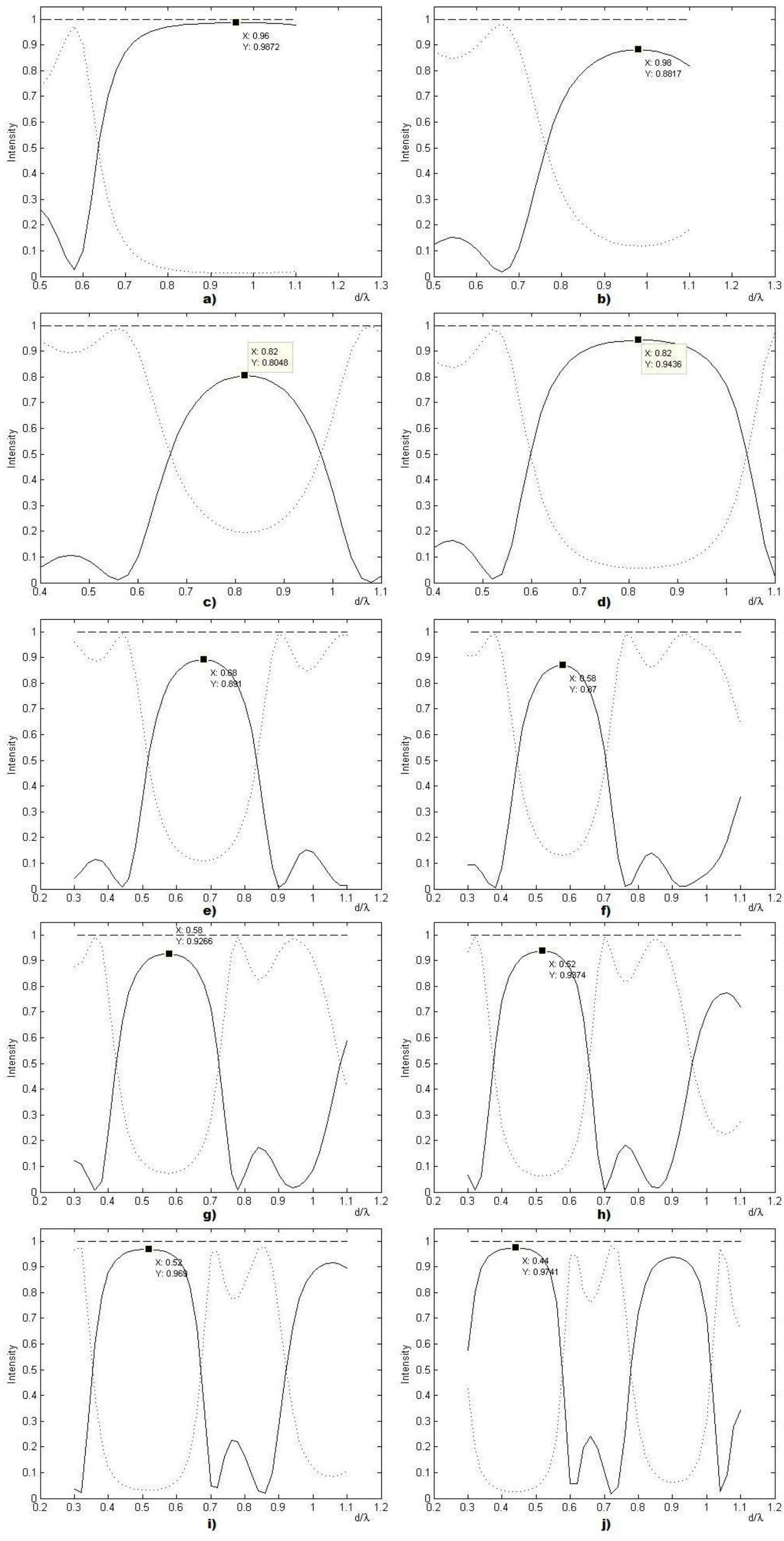

Figure 5. Reflectance (solid curve) and transmittance (dotted curve) intensities for a sinusoidal profile following a process of optimization.

5 CONCLUSIONS

The method for calculating reflectance and transmittance coefficients showed to be a good tool in the characterization of the proposed profiles. Interesting behaviors obtained with the changes in the different parameters that defined the profiles where observed that help to improve the characteristics of the filters. However it is a hard task to achieve good parameters since many conditions should be satisfied not only regardig the mathematical implementation but also from the practical point of view for the realization of the filters.

The time elapsed in computation (in general over 30 minutes) makes it difficult to evaluate some other interesting topics as the reflectance both as a function of time –that would help study light dispersion– or in terms of varying incidence angles –which would be useful for the design of light couplers. The implementation for real time computing by now seems to be far away.

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