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STUDY OF ANALYTICAL MODELS OF THE MECHANICAL BEHAVIOR OF POLYMER-MODIFIED CONCRETE

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Abstract. *Polymer modification of mortar and concrete is a widely used technique in order to improve their durability properties. Hitherto, the main application fields of such materials are repair and restoration of buildings. However, due to the constant increment of service life requirements and the cost efficiency, polymer modified concrete (PCC) is also used for construction purposes. Therefore, there is a demand for studying the mechanical properties of PCC and entitative differences compared to conventional concrete (CC). It is significant to investigate whether all the assumed hypotheses and existing analytical formulations about CC are also valid for PCC. In the present study, analytical models available in the literature are evaluated. These models are used for estimating mechanical properties of concrete. The investigated property in this study is the modulus of elasticity, which is estimated with respect to the value of compressive strength. One existing database was extended and adapted for polymer-modified concrete mixtures along with their experimentally measured mechanical properties. Based on the indexed data a comparison between model predictions and experiments was conducted by calculation of forecast errors.*

1 INTRODUCTION

Polymer-modified concrete (PCC) has been a widely used material in repair and restoration of buildings. Due to the constant increment of service life requirements and the cost efficiency, it is more and more utilized for construction purposes. This process requires the pronounced investigation of the mechanical performance of polymer-modified concrete. The modification in the composition of the mixture as a result of polymer addition affects the behavior of the concrete in its fresh and hardened state. Therefore, it is important to estimate precisely the fundamental mechanical properties of this material.

The characterization of the mechanical performance of different kinds of concrete has been investigated for many decades (e.g. [1], [2], [3], [4], [5]). Hitherto, numerous constitutive relations for the prediction of properties such as the Young's modulus, the tensile strength or stress-strain curves have been developed. Nonetheless, their fields of application are limited in most cases to conventional concrete (CC) [6]. One crucial question is whether or not the behavior of PCC can be translated to the properties of conventional concrete. More particularly, it is significant to investigate if the assumed hypotheses and existing analytical formulations are applicable for polymer-modified concrete. Present models may not take into account the complexity of PCC and, therefore, may lead to inaccurate predictions of fundamental mechanical properties. Thus, it needs to be clarified if existing design codes should be modified for the application to new construction materials.

Within this paper, analytical expressions for estimating the modulus of elasticity (MOE) were investigated. The so-called Young's modulus of concrete directly affects the stiffness and deformation behavior of structural components. It needs to be known for both the determination of deflections in structures for requirements in serviceability and to calculate prestressing forces in prestressed concretes [7],[8]. There are various factors affecting the Young's modulus of concrete. The most important ones are classified in Figure 1.

To compare the goodness-of-fit of those models, an existing database [9] was used and extended for data from polymer-modified concretes. The comparison between model calculations and experimental data was done by using different quality criteria.

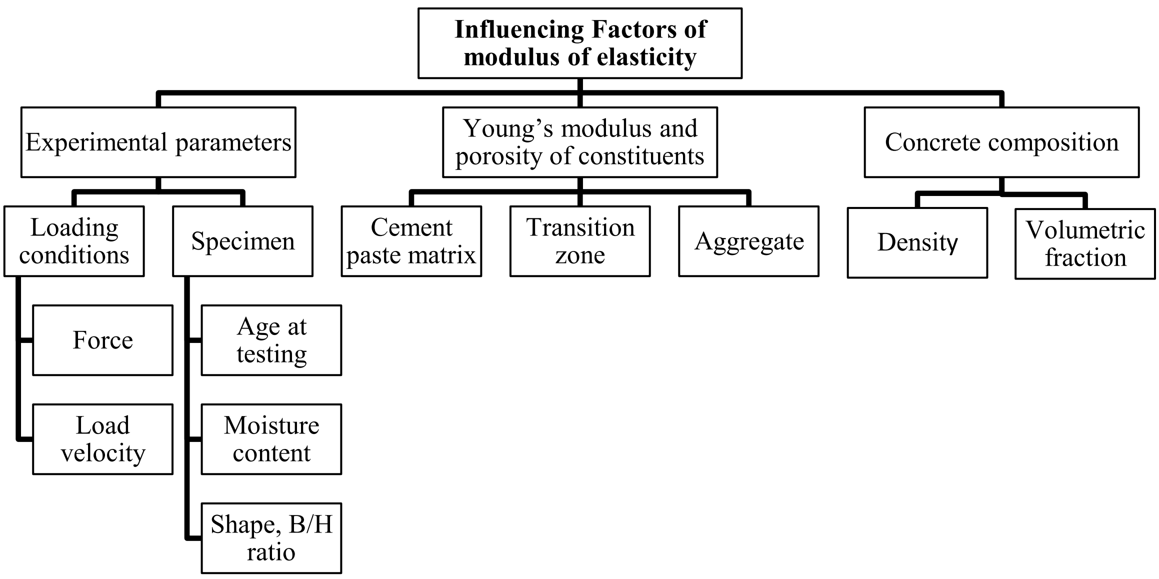

Figure 1: Most important factors affecting the modulus of elasticity of concrete

2 MATERIAL PHENOMENA OF PCC

Polymer-modified concretes exhibit different behavior compared to conventional concretes. The addition of polymers to the concrete mixture varies from 5 % up to 20 % of the cement content in which various types of polymers can be used. For concretes modified with styrene-butadiene and styrene-acrylic acid ester in form of latex dispersion or redispersible powders most studies have been conducted [10]. Besides their chemical nature, the polymers vary regarding their consistency as well as their minimum film formation temperature. Those factors also have a large influence on the properties of fresh and hardened PCC.

A lot of research has been conducted with regard to observations of changes in the microstructure of cementitious systems [11], [12], [13]. Polymer and cement phases interpenetrate each other [14], and they form together the binder matrix. A continuously increasing number of publications considers the changed mechanical behavior of PCC, e.g. [15], [16], [17], [18]. The Young's modulus as well as the compressive strength normally decrease, the material shows a more ductile behavior, and the viscous properties are more conspicuous, resulting in large time-dependent deformations [19–21], [20], [21].

One question is the application of analytical formulas describing the mechanical performance of CC to PCC. KEITEL evaluated the use of existing creep models developed for CC representing PCC. The author compared calculated creep strains to experimental data of PCC, and optimized the model parameters to minimize the differences between them [22], [23]. The adaptation of design codes, aiming at prediction of the Young's modulus of concretes, for the use of PCC is not studied yet.

3 SURVEY OF EXPERIMENTAL DATA AND STRUCTURAL DESGIN CODES

3.1 Existing databases

A large variety of experimental results of mechanical laboratory tests concerning conventional concretes are available in the open literature. LIM et al. [9], ASLANI et al. [24], and CRAEYE et al. [6] provide an overview about existing analytical formulations for different types of concrete and constructed databases to summarize information about mix-design and properties of concretes used in numerous experimental studies.

LIM et al. [9] assembled a database from 209 experimental studies and summarized 4353 datasets. The results were sorted into different groups according to the type of concrete (normal-weight concrete (NWC) and light-weight concrete (LWC)) and the cross-sectional shape of specimen (square or circular). CRAEYE et al. [6] provide an overall view on the mechanical performance of self-compacting concrete (SCC). The database contains results of more than 250 publications. ASLANI et al. [24] assembled around 250 mixtures for a database of CC and SCC and analyzed the mechanical properties for those kinds of concrete.

3.3 Extension of an existing database

The database by LIM et al. [9], which was chosen to be the most complete one, was further expanded by adding more recent experimental results about PCC. A literature study was carried out. The additional data stems from several publications about the mechanical performance of polymer-modified concrete and related laboratory experiments. Entirely, the database contains results of more than 25 papers. It includes information with regard to mix-design, fresh and hardened properties of PCC. The structure of the database is shown in Figure 2. It should be noted that in some of the datasets details were not available from the source documents. To guarantee comparability between different concretes and for a consistent treatment of the test results inside the database, only datasets from concretes with a circular cross-section were used.

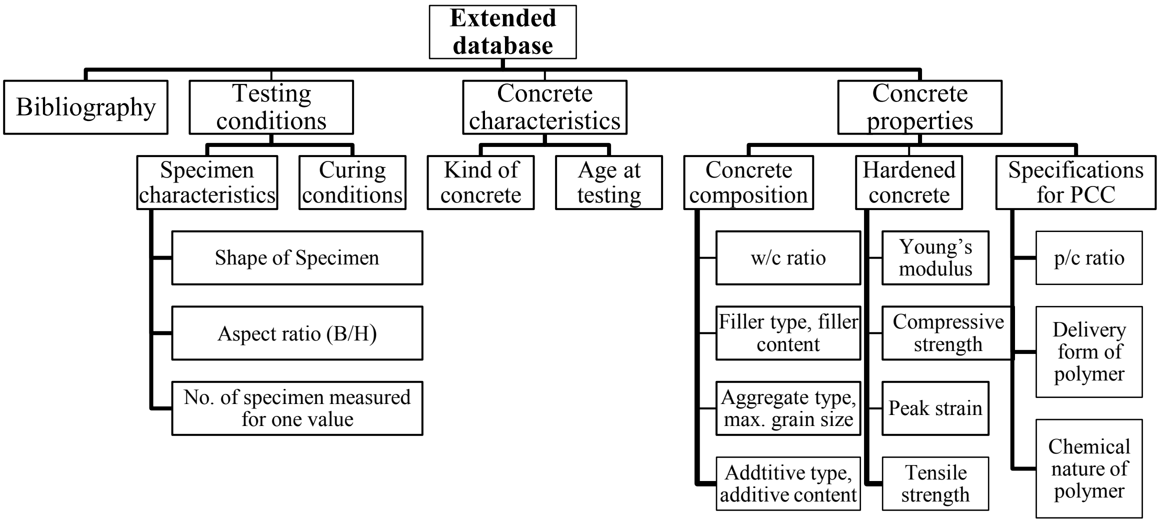

Figure 2: Structure of the database including properties related to PCC

In the database of the polymer-modified concretes, the water-to-cement ratio varied from 0.23 to 0.84, the density of the hardened concrete from 1560 to 2440 kg/m^3, the experimentally determined Young's modulus from 10,000 to 39,500 kN, and the concrete compressive strength from 14 to 68 kN. Figure 3 shows the distribution of the compressive strength of the normal and light weight concrete specimens, and of the polymer-modified concrete specimens. Apparently, the collected PCC specimen have a compressive strength around 40 kN whereas the compressive strengths of the conventional concretes are slightly lower according to this database.

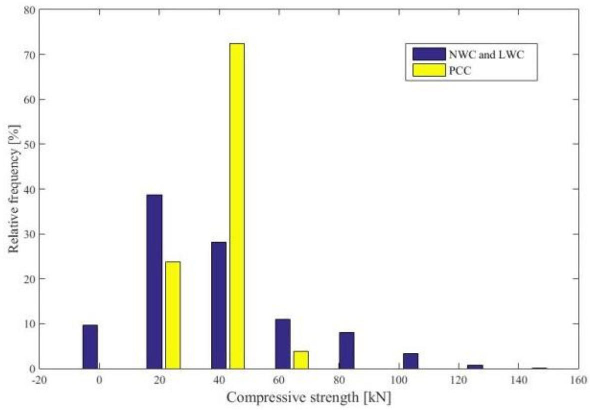

Figure 3: Relative frequencies of compressive strength values of conventional concretes (NWC and LWC) and polymer-modified concretes (PCC) from laboratory experiments

Furthermore, the database was used to investigate some polymer-specific properties. The polymer-to-cement ratio used for modification of the concretes varies between 0.01 and 0.30. So, Figure 4 shows the occurrence of the four most often polymers used for modification purposes when comparing the datasets of the database. As illustrated, most research results are available about concretes modified with styrene-butadiene.

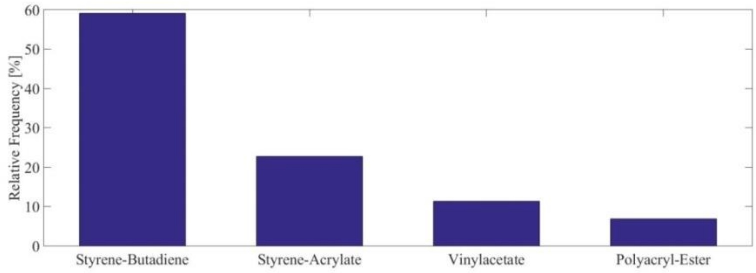

Figure 4: Relative frequencies of polymers used for modification of cementitious systems

3.2 Limitations of the database

Using experimental results from different published sources include problems because often there are information missing regarding the exact composition of the concrete mixtures and the testing regime, respectively. Furthermore, the extraction of relevant results in some cases is difficult because of an incompleteness of the published values. Partly, the data are presented in diagrams and have to be extrapolated from the graphs [24].

Such databases contain datasets from different laboratory experiments and, thus, comprise a great variety of cementitious material. The experimentally determined mechanical properties depend on many influencing factors due to the complex behavior of concretes. The indexed parameters differ from one investigation to another. There are differences in the material composition (aggregate size, type and amount of cement, aggregate, or additive, respectively), the age of the specimen, curing conditions, testing facility, and the measurement equipment. Loading rates, for example, cover several orders of magnitude (from very slow tests up to tests within a few seconds). Therefore, the gathered experimental data scatter significantly, which has to be considered during the interpretation of the results.

3.4 Models for the modulus of elasticity

The modulus of elasticity for concrete can be calculated with empirical equations. The models were developed by several institutions. Many researchers have provided additional recommendations based on evaluation of experimental works. Most of the formulas relate the initial or secant modulus to the compressive strength of concrete. Due to the large influence of the concrete density and the type of aggregates used in the concrete mixtures, some models consider those parameters with correction factors. However, because of many influencing factors, the calculated value never represents the actual value of the modulus of elasticity [8].

Within this paper, existing analytical models were separated into three groups. The formulations in the first group predict the MOE only with the compressive strength of concrete as an input parameter. The second group additionally takes into account the density of the hardened concrete. Many researchers have recognized the determining effect of the aggregate properties in predicting MOE of concrete [24], and, thus, a third class of equations was distinguished, including parameters for different types of aggregates. A more detailed explanation about the models is given in the original publications. The models used in this study are summarized in Table 1, Appendix 1.

4 RESULTS AND DISCUSSION

4.1 Comparison of model outputs and experimental data

The experimental data from the database were used to evaluate how good the models developed for conventional concretes predict the modulus of elasticity for polymer-modified concretes. In a first step, the quotient between the mean values of both the calculated Young's modulus and the experimentally determined Young's modulus were calculated for every model. The calculation was done on the basis of the same value of the compressive strength. The results are shown in Table 2. The closer the values converge to 1, the better fits the model the experiments. The comparison was done for all three types of concrete.

Table 1: Comparison between calculated and experimental results for the Young's modulus

Input parameter	f_c					f_c **and** γ				f_c **and** γ **and** K_1		
Model	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
NWC	1.05	0.99	1.21	1.24	1.16	1.08	1.02	0.53	0.75	0.99	1.67	1.21
LWC	1.61	1.92	2.34	2.14	1.99	0.80	1.16	0.46	0.92	1.04	0.86	1.15
PCC	1.14	1.09	1.34	**1.35**	1.26	1.12	1.14	**0.59**	0.85	**1.00**	**1.00**	1.33

It can be seen that the values of PCC are almost in the same range like the values of NWC. That means that the models developed for CC are able to predict the Young's modulus of PCC. The mean values for LWC are much higher which indicates that the models overestimate the property for this type of concrete.

There are differences between the prediction qualities of the different models. For PCC, the best estimation is derived by application of both the AASHTO and the BAALBAKI model which take into account the type of aggregate. In case of PCC, it therefore seems to be necessary to use a model with more than only one input parameter. For NWC, it is already sufficient to apply the models suggested by almost all national design codes that only use the compressive strength for estimation.

4.2 Quality criteria

To prove the results from the previous section, two other types of quality criteria were calculated to evaluate the goodness-of-fit of the models. The root-mean-squared errors (RMSE) as well as the mean absolute percentage errors (MAPE) were determined. The lower the calculated value of both criteria is, the better is the prediction capability of the model. Table 3 shows the results of the calculations. Due to the bad performance of the models in case of light-weight concretes, only NWC and PCC were considered in this section.

It can be seen that both the calculation of RMSE and MAPE and the direct comparison between calculated and experimental results are almost comparable. Hence, also the use of models that consider the type of aggregates is recommended for application to PCC because results with the least errors can be achieved by applying those types of models. When the forecast errors between the model outputs for NWC and PCC are compared, it becomes obvious that there are almost no differences. In case of the first group of models, the model predictions are better for NWC. Otherwise, if the aggregate type is considered, the forecast errors are similar for both types of concrete.

Table 2: Forecast errors for estimating the Young's modulus of concrete by different models

Input parameter	f_c					f_c and γ				f_c and γ and K_1		
Model	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
NWC												
RMSE ($\cdot 10^{-3}$)	4.99	4.93	6.53	8.85	6.84	5.10	5.36	17.77	4.79	6.92	4.89	5.15
MAPE [%]	13.5	13.3	22.5	25.4	19.0	12.9	13.4	46.4	12.2	17.8	10.4	11.2
PCC												
RMSE ($\cdot 10^{-3}$)	4.51	3.44	9.51	9.43	7.34	3.94	4.62	8.26	1.07	4.41	2.88	2.71
MAPE [%]	19.5	16.0	34.1	**35.3**	27.2	13.3	14.1	**40.9**	12.1	16.0	**11.3**	**11.2**

5 CONCLUSION

The main focus of existing studies about polymer-modified concrete has been related to changes in the microstructure, the mix-design, and durability since it is a relatively new type of concrete. The application of models describing the mechanical behaviour of PCC has not been investigated yet. Nevertheless, in most papers information about the compressive strength, and in some cases, about the modulus of elasticity and the tensile strength are published. So, an existing database, developed for conventional concrete, was extended by addition of datasets for PCC. Results from more than 25 papers were assembled. The database was used to compare experimental data with the results of empirical formulations for estimating the Young's modulus of concrete.

It can be concluded that the models for estimating the modulus of elasticity can be applied to PCC, especially those which consider the type of aggregate. The experimentally determined data of PCC seem to confirm the model predictions and fit well into the bandwidth expected for the secant modulus of elasticity in general. Even the assumed lower Young's modulus of PCC due to the role of polymers in the matrix and the formation of sliding planes inside the microstructure [18] does not deteriorate the capability of the models developed for CC for their application to PCC. However, it should not be neglected that the scatter of the experimentally determined Young's modulus for concretes which have the same compressive strength can be enormous. Hence, a precise determination of the Young's modulus is only possible with laboratory experiments, conduction of static tests, or with ultrasonic measurement methods.

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APPENDIX 1

Table 3: Models for the modulus of elasticity developed for conventional concretes

No.	Reference	Model for modulus of elasticity	Input parameter
1	ACI 318 (2008) [25]	$E_c = 4730\sqrt{f_c}$	f_c [1]
2	ACI363 (1992) [26]	$E_c = 3320 \sqrt{f_c} + 6890$	f_c
3	EHE (2010) [27]	$E_c = 10000 \cdot \sqrt[3]{f_c}$	f_c
4	NBR 6118 (2014) [28]	$E_c = 5600 \sqrt{f_c}$	f_c
5	HUESTE (2004) [29]	$E_c = 5230 \sqrt{f_c}$	f_c
6	AHMAD and SHAH (1985) [24]	$E_c = 3.38 \cdot 10^{-5} \cdot \gamma^{2.5} \left(\sqrt{f_c}\right)^{0.65}$	f_c and γ [2]
7	JOBSE and MOUSTAFA (1984) [24]	$E_c = 0.103 \cdot \gamma^{1.5} \left(\sqrt{f_c}\right)^{0.5}$	f_c and γ
8	COOK (1989) [30]	$E_c = 3.22 \cdot 10^{-5} \cdot \gamma^{2.5} \left(\sqrt{f_c}\right)^{0.315}$	f_c and γ
9	LIM (2014) [9]	$E_c = 4400 \sqrt{f_c} \left(\frac{\gamma}{2400}\right)^{1.4}$	f_c and γ
10	AASHTO (2006) [24]	$E_c = 0.043 \; K_1 \cdot \gamma^{1.5} \cdot \sqrt{f_c}$	f_c and γ and K_1 [3]
11	BAALBAKI, W. (1997) [24]	$E_c = K_1 + 0.2 f_c$	f_c and γ and K_1
12	CEB-FIP (1993) [31]	$E = 21500 \cdot K_1 \sqrt[3]{\frac{f_c}{10}}$	f_c and γ and K_1

[1] Cylinder compressive strength of concrete (standard specimens) [kN]
[2] Unit weight of concrete [kg/m^3]
[3] Correction factor for source of aggregate, specific values for specific types aggregates