

CALIBRATION OF CYCLIC CONSTITUTIVE MODELS FOR SOILS BY OSCILLATING FUNCTIONS

M. Alalade*, B. Kafle, F. Wuttke and T. Lahmer

**Bauhaus Universität Weimar
Department of Civil Engineering
E-mail: muyiwa.alalade@uni-weimar.de*

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Abstract. *In order to minimize the probability of foundation failure resulting from cyclic action on structures, researchers have developed various constitutive models to simulate the foundation response and soil interaction as a result of these complex cyclic loads. The efficiency and effectiveness of these model is majorly influenced by the cyclic constitutive parameters. Although a lot of research is being carried out on these relatively new models, little or no details exist in literature about the model based identification of the cyclic constitutive parameters. This could be attributed to the difficulties and complexities of the inverse modeling of such complex phenomena. A variety of optimization strategies are available for the solution of the sum of least-squares problems as usually done in the field of model calibration. However for the back analysis (calibration) of the soil response to oscillatory load functions, this paper gives insight into the model calibration challenges and also puts forward a method for the inverse modeling of cyclic loaded foundation response such that high quality solutions are obtained with minimum computational effort. Therefore model responses are produced which adequately describes what would otherwise be experienced in the laboratory or field.*

1 INTRODUCTION

The increased awareness in the impact of cyclic actions from natural and man-made sources on a structure during its lifespan necessitates understanding the complex soil structure interaction during these cyclic loads. The stresses induced by these cyclic loads lead to failure patterns such as flow-type failure (liquefaction), cyclic mobility or plastic strain accumulation. The failure type experienced depends on the initial stress state and degree of stress reversal as observed by [1]. In order to design structures such that the effects of these stresses are minimized, various constitutive models based on different formulations such as –plasticity, elasto-plasticity, hypoplasticity and so on, to describe the soil structure interaction during complex cyclic loading on dry, saturated and unsaturated soils have been developed.

The last three decades heralded the development and extension of many constitutive models some of which includes the elasto-plastic strain hardening law model by [2] for monotonic loads which served as a foundation for many other models such as the model by [3] for cyclic loads and model by [4], [5] for monotonic and cyclic loaded footings on unsaturated soil which takes into account the effect of matric suction on the bearing capacity of the soil as observed by [6]. The models in [3] and [5] for cyclic loading is capable of simulating the foundation response under different loading conditions and soil states. However, the quality of the model's response is strongly influenced by certain constitutive parameters, thus in order to apply the models for the actualization of economical and efficient structural design, we have to bridge the gap between model simulation and experimental response. This is achieved by the calibration and optimization of the models' sensitive 'cyclic constitutive parameters' with experimental results.

The importance of these constitutive models cannot be over-emphasized, especially when considering the ease with which it can be incorporated into finite element software for structural analysis, structural design and structural health monitoring as well, therefore in order to yield reliable model responses an in-depth knowledge on the cyclic parameter behavior is required in addition to selecting an efficient optimization strategy.

2 THEORY

2.1 Mathematical Model

The elasto-plastic strain hardening model proposed by [3] for response to cyclic loading on dry soil is an extension of the model proposed by [2] which was developed for monotonic loading. In order to simulate cyclic loading, the model in [3] permits the formation of plastic strains inside the bounding surface. The magnitude of plastic strains induced is a function of the distance between the stress point and the image point I_p on the bounding surface, the memory parameters (ρ_c and ρ_k) and the cyclic constitutive parameters expressed in the Φ term in Equation 1. The parameters $\Lambda(Q_I)$ and $\frac{\delta g}{\delta Q}(Q_I)$ are calculated on the image point and not on the current stress point as would be the case for monotonic loading

$$dq^{pl} = \Lambda(Q_I) \Phi(\delta, \rho_k) \frac{\delta g}{\delta Q}(Q_I) \quad (1)$$

The Φ term which is a function of δ and ρ_k is a diagonal matrix (Equation 2) which acts as a weighting function during cyclic loading. Larger values of δ leads to small diagonal terms and

also smaller plastic displacements [3]

$$\Phi = \begin{bmatrix} \exp\left(-\left(\varsigma_\eta \sqrt{\frac{\delta}{\xi} + \kappa_\eta \rho_k}\right)\right) & 0 & 0 \\ 0 & \exp\left(-\left(\varsigma_\epsilon \sqrt{\frac{\delta}{\xi} + \kappa_\epsilon \rho_k}\right)\right) & 0 \\ 0 & 0 & \exp\left(-\left(\varsigma_\zeta \sqrt{\frac{\delta}{\xi} + \kappa_\zeta \rho_k}\right)\right) \end{bmatrix} \quad (2)$$

Plastic strain caused by cyclic loading is sensitive to the constitutive parameters ς_i and κ_i (where $i = \eta, \epsilon, \zeta$) in the Φ matrix. These parameters regulate the rate at which shakedown occurs in the model and also the the coupling between the horizontal loading and the corresponding vertical displacement at constant vertical force.

The application of this model to unsaturated soil condition was made possible by its extension in [5] which is expressed in Equation 3 where it was observed according to the findings of [7] and [8] that not only the maximum vertical force (V_m) and the initial soil stiffness (R_0) are sensitive to matric suction but also the cyclic constitutive parameters, thus simulating the response that would be expected for unsaturated soil as compared to dry or saturated soils. This model can be seen in Equation 3. Though similar to Equation 1 the parameters are a function of the matric suction ψ

$$dQ^{pl}(\psi) = \Lambda_I(\psi) \Phi(\delta, \rho_k, \psi) \left(\frac{\delta g(\psi)}{\delta Q(\psi)} \right)_I \quad (3)$$

2.2 Calibration and optimization

The quality of any prognosis depends on the quality of the identified cyclic constitutive parameters. Besides accuracy, efficiency is also an important issue as dynamic non-linear models tend to be complex and time consuming, particularly during calibration. Therefore it was necessary to study the cyclic constitutive parameter dependency using contour lines before selecting an optimization strategy. A strong dependency between some parameters was observed and is expressed in Figure 1. The presence of many valleys (indicating minima) can be traced to the structure of the Φ matrix which has addition operations in each of the diagonal terms as seen in Equation 2. Thus different pairs of cyclic constitutive parameters in each diagonal term in the Φ matrix can yield similar results with regards to model response.

In order to minimize now regularized sum of squared error cost functions, the Nelder-Mead method ([10]), which is heuristic based, deterministic and known to be robust with respect to noisy cost function is applied. The objective function being minimized is the sum of squared error with respect to the vertical cyclic displacement, η [3]

$$C'_f(p) = \sum_i^n (\eta_{(t_i,p)}^{mod} - \eta_{(t_i)}^{exp})^2 + \Gamma \sum_{k=1}^4 (\Psi_{(k)}^0 - \Psi_{(k)}^{mod})^2 \quad (4)$$

where:

- C'_f is the cost function to be minimized

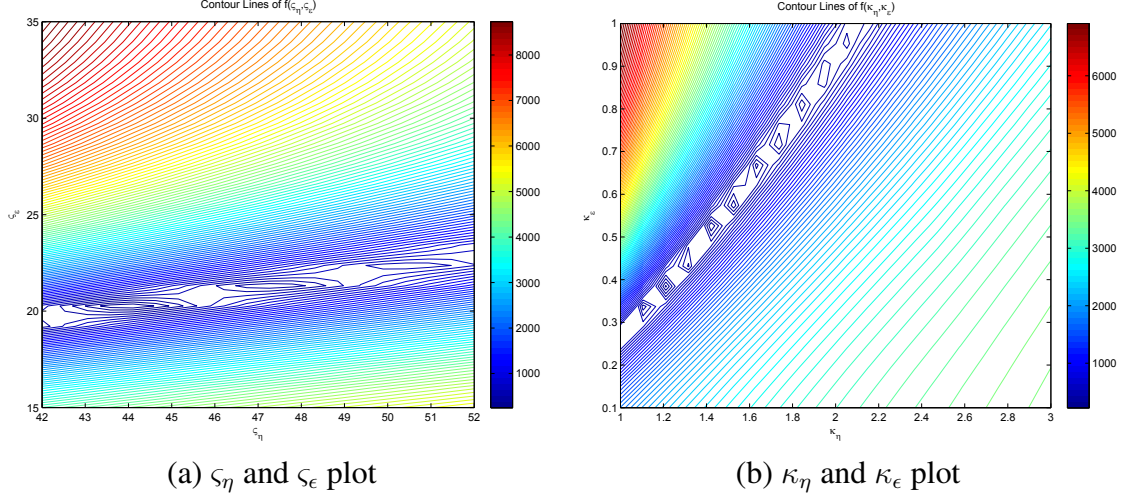


Figure 1: Dependencies between cyclic constitutive parameters [9]

- η^{mod} is the vertical component of the generalized cyclic plastic displacement vector calculated by the model
- η^{exp} is the vertical component of the generalized cyclic plastic displacement vector obtained from the experiment
- p is a vector of $\Lambda_I(\psi)$, $\Phi(\delta, \rho_k, \psi)$ and $\left(\frac{\delta g(\psi)}{\delta Q(\psi)} \right)_I$
- t_i is the time step at each point
- n is the total number of data points
- Γ is a scaling factor for the penalty term
- Ψ^0 is a vector of a priori cyclic constitutive parameters
- Ψ^{mod} is a vector of calculated cyclic constitutive parameters
- k is the total number of cyclic constitutive parameters

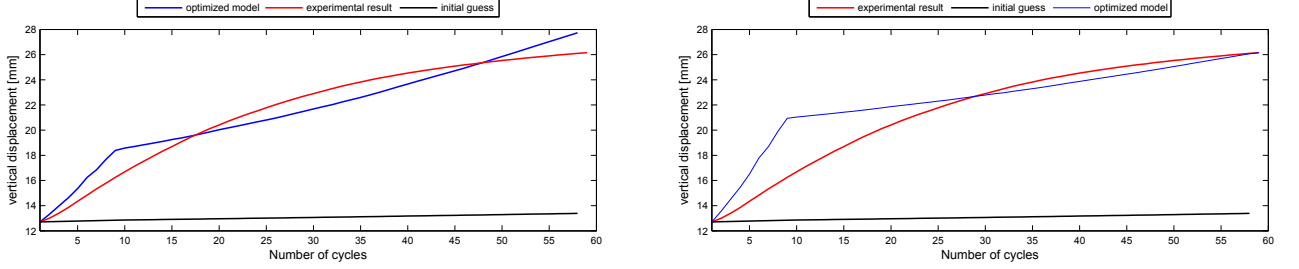
3 RESULTS

Model parameter identification was carried out by calibrating the model to experimental data obtained from [5]. Before the model calibration, it was necessary to filter out measurement errors from the experimental data. The model parameter identification was carried out using the Nelder-Mead algorithm on dry and unsaturated soils, with outcomes shown in Table 1 and Figure 2 to Figure 3.

It is worth noting that as a result of the strong dependency between the cyclic constitutive parameters and the application of a deterministic local optimization strategy, the values used to initiate the optimization plays a major role in the computational time required and the quality of the output result obtained. Although values from literature (values published in [11]) were first used to initiate the optimization process, the quality of fit was not satisfactory, thus other randomly generated values were used as initial values until a best fit was attained.

Table 1: Parameters obtained after model optimization with and without regularization

State	ψ [kPa]	V_m [kN]	R_0 [kN/mm]	ζ_η	κ_η	ζ_ϵ	κ_ϵ	F.value	Iter.
Sat. ($\Gamma = 0$)	0	1.12	0.252	54.3857	0.3435	-59.3968	74.9093	50.77	482
Sat. ($\Gamma \neq 0$)	0	1.12	0.252	52.2103	0.1825	27	55	47.11	328
Unsat. ($\Gamma = 0$)	2.1	6.4	0.822	7.1696	3.3187	12.9429	109.6090	1.143	121



(a) accumulated plastic displacement plot ($\Gamma = 0$) (b) accumulated plastic displacement plot ($\Gamma \neq 0$)

Figure 2: Comparison between non-regularized and regularized optimized model response for saturated soil

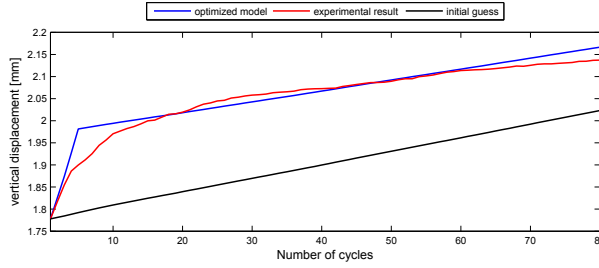


Figure 3: Comparison between the initial model response, experimental response and optimized model response for unsaturated soil

4 CONCLUSION

As observed in Figure 2 and Figure 3 and also corroborated in literature, the unsaturated soil yields lower plastic displacements when compared to other soil states, in addition to this and with respect identifying the model parameters it can be concluded that

- Measurement error (noise) affects quality of fit during calibration and thus have to be filtered out before parameter identification
- The initial guess is a key factor in determining the outcome (quality of parameters obtained) and efficiency of the the inverse modeling computation.
- Though the Nelder–Mead algorithm may converge to a local minima, the process of using random initial values and comparing the quality of fit shows the application of this algorithm to find the global minima with a high probability.
- Better quality fit was observed for unsaturated soil condition with minimal computational effort. This can be attributed to the fact that shakedown occurs earlier and as such the effect of the slight reduction in the accumulated plastic displacement during unloading by soil rebound which the model does not account for is minimized.

- With regularization, calibration yields more reliable results with minimal computational effort for these complex dynamic soil models as seen in Table 1.

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