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# THE DISCRETE MODEL OF CRACKS ACCORDING TO BORCZ'S THEORY IN ORDER TO CALCULATE THE DEFLECTIONS OF BENDING REINFORCED CONCRETE BEAMS

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**Abstract.**
*In the design of the reinforced concrete beams loaded by the bending moment, it is assumed that the structure can be used at a level of load, that there are local discontinuities - cracks. Designing the element demands checking two limit states of construction, load capacity and usability. Limit states usability include also the deflection of the element.*
*Deflections in the reinforced concrete beams with cracks are based on actual rigidity of the element. After cracking there is a local change in rigidity of the beam. The rigidity is variable in the element's length and due to the heterogeneous structure of concrete, it is not possible to clearly describe those changes. Most standards of testing methods tend to simplify the calculations and take the average value of the beam's rigidity on its entire length. The rigidity depends on the level of the maximal load of the beam. Experimental researches verify the value by inserting the coefficients into the formulas used in the theory of elasticity. The researches describe the changes in rigidity in the beam's length more precisely. The authors take into consideration the change of rigidity, depending on the level of maximum load (continuum models), or localize the changes in rigidity in the area of the cracks (discrete models).*
*This paper presents one of the discrete models. It is distinguished by the fact that the left side of the differential equation, that depends on the rigidity, is constant, and all effects associated with the scratches are taken as the external load and placed on the right side of the equation. This allows to generalize the description.*
*The paper presents a particular integral of the differential equation, which allow analyzing the displacement and vibration for different rigidity of the silo's walls, the flow rate and type of the flowing material.*

## 1 INTRODUCTION

In construction statics, beam deflections are inversely proportional to their rigidity. The dependence, though, is true in the case of material, the rigidity of which does not depend on the load level and des not change in time. Such a dependence is, for instance, to a large extent true for the steel beams.

In the case of another kind of material, such as concrete or reinforced concrete, the rigidity is reduced as a result of loading of the beam. Initially, the extended sections get plasticised, and finally, in the extension stress zone cracks appear. In the cracked sections, the beam rigidity is considerably decreased. It causes a noticeable, disproportionate deflection increase.

The computational methods most frequently include it directly through implementing a function changing the rigidity into the formula (the continuum method). In the presented method, always a constant primal stiffnes of the element is assumed, and it enables calculating the increase of deflections caused by the cracks in the structure and rheological effects (the discrete method).

In the following chapters, the assumptions made for the calculations and the method of their practical use is presented.

## 2. MODELS OF CALCULATING RIGIDITY IN THE REINFORCED CONCRETE BEAMS

In the construction statics, the differential equation of a deflected axis is derived from the geometrical dependences and balance equations. For small shifts, the differential equation of the deflection line of the bent beam is:

$$\frac{1}{L^2}\frac{\partial^2}{\partial \xi^2}\left[\frac{1}{L}EI(\xi,t)\frac{\partial^2 v(x,t)}{\partial \xi^2}\right] = p(\xi) \tag{1}$$

Deflections of cracked reinforced concrete beams depend on factual element's rigidity. After cracking a local change of rigidity occurs. The rigidity changes along the element length. For reasons of inhomogeneity of structure of concrete an explicit description of these changes is impossible. Most of the codes' methods are aimed at simplicity of calculations and assume mean value of rigidity in whole length of a beam. It depends on load level of a beam. The experimental studies verify the computational value of rigidity by introducing coefficients to the formulae applying in theory of elasticity. The investigative theories describe changes of rigidity along a beam length in more precise way. Authors take rigidity changes into account depending on load level (continual models) or localize rigidity changes near by cracks (discrete models) – figure 1.

According to EC 2 and most of the Authors the rigidity change of bending beam is taken into account with the term describing rigidity:

$$\frac{\partial^2}{\partial x^2}\left[EJ_{I,II}\frac{\partial^2 v(\xi,t)}{\partial x^2}\right]=p(\xi) \tag{2}$$

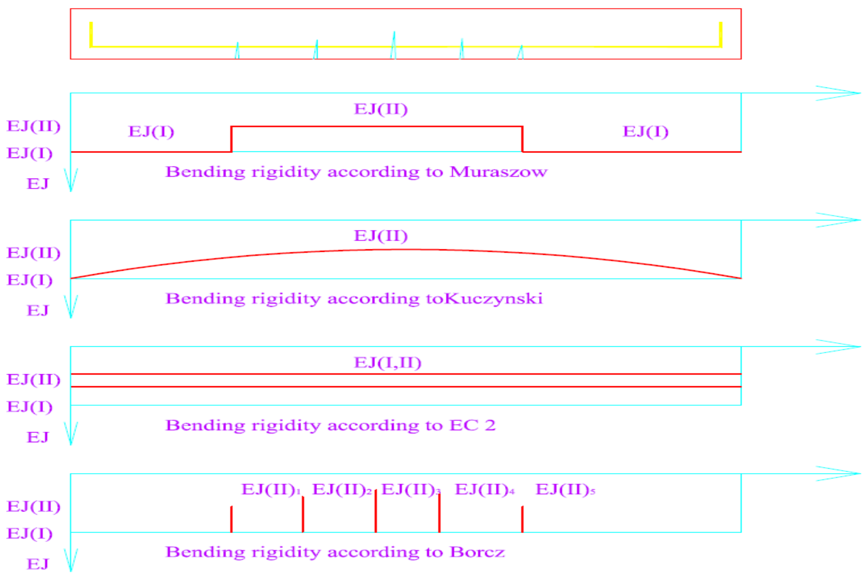


Fig. 1. The beam's rigidity according to the different Authors and EC2

In terms of quality, different is a proposal implied by the Borcz theory, according to which the left side of the differential equation does not depend on the maximum load of the beam. The form of the left side is identical with the one of the classical elasticity theory (the linearly elastic material and the infinite strength). The model takes into account both the permanent deformations in the material, the rheological deformations, and the maximum load of a section. The model is presented in chapter 3.

## 3. MODELS OF CALCULATING RIGIDITY IN THE REINFORCED CONCRETE BEAMS ACCORDING TO BORCZ

### 3.1. The bent element

According to Borcz the rigidity of beam after cracking is the same as before. Each effects connected with cracking are assumed as external load.

$$\frac{\partial^2}{\partial x^2}\left[EJ_{I}\frac{\partial^2 v(\xi)}{\partial x^2}\right]=p(\xi)+\sum_r r_i\delta,_{\xi\xi}(\xi-\zeta_r) \tag{3}$$

$r_i$ – pitch angle in the area of the crack, $\delta,_{\xi\xi}(\xi-\zeta_r)$ – the second Dirac delta derivative.

Part of a deflection resulting from the existing cracks is expressed on the right side of the equation (3) as the change of angular displacement in the place of crack occurrence. In the place of crack, elastic strains and plastic deformations are localized. The model has been to some extent experimentally verified.

In the model of Borcz general solution of the equation (3) does not depend on the occurrence of cracks in element. It allows to apply the solution of classic theory of elasticity for static analysis of reinforced concrete structure with cracks.

After fourfold integration, equation (3) will assume the form of a polynomial,the constant results of which result from the boundary conditions:

$$v(\xi)=\frac{1}{L}\delta_0+\xi\varphi_0-\frac{\xi^2 L}{2EJ}M_0-\frac{\xi^3 L^2}{6EJ}T_0+\bar{v}(\xi)+\sum_i r_i(\xi-\zeta_i)h(\xi-\zeta_i) \tag{4}$$

$v(\xi)$ - special integral of the differential equation $h(\xi-\zeta_i)$ – Heaviside's function

The curve formula:

$$v,_{\xi\xi}(\xi)=-\frac{L}{EJ}M_0-\frac{\xi L^2}{EJ}T_0+\overline{v,_{\xi\xi}}(\xi)+\sum_i\left\{\left[-r_{0i}+r_{1i}v,_{\xi\xi}\left(\zeta_i^-\right)\right]\delta\left(\xi-\zeta_i^-\right)\right\}, \tag{5}$$

and after substitution with the coordinates of the crack:

$$v,_{\xi\xi}\left(\zeta_i^-\right)=-\frac{L}{EJ}M_0-\frac{\xi L^2}{EJ}T_0+\overline{v,_{\xi\xi}}\left(\zeta_i^-\right), \tag{6}$$

After implementation of equation (6) into equation (3):

$$\begin{aligned}v(\xi)=&\frac{1}{L}\delta_0+\xi\varphi_0+\left[-\frac{\xi^2 L}{2EJ}-\sum_i r_{1i}\frac{L}{EJ}(\xi-\zeta_i)h(\xi-\zeta_i)\right]M_0\\&+\left[-\frac{\xi^3 L^2}{6EJ}-\sum_i r_{1i}\zeta_i\frac{L^2}{EJ}(\xi-\zeta_i)h(\xi-\zeta_i)\right]T_0\\&+\left[\bar{v}(\xi)+\sum_i r_{1i}\bar{v},_{\xi\xi}(\xi-\zeta_i)h(\xi-\zeta_i)+\sum_i -r_{0i}(\xi-\zeta_i)h(\xi-\zeta_i)\right]\end{aligned} \tag{7}$$

Differentiating the shift resulting from it (7) in relation to $\xi$ one gets vector**{u($\xi$)}**. The solution canalso be expressed in a form of a matrix as a sum of the solution of the homogenous and the cracked structure model:

$$\{\mathbf{u}(\xi)\} = [\mathbf{F}_E(\xi) + \mathbf{F}_{res}(\xi)]\ \{\mathbf{u}_0\} + \{\mathbf{u}'(\xi)\} \tag{8}$$

$$[\mathbf{F}_E(\xi)]=\begin{bmatrix}\frac{1}{L} & \xi & \frac{-\xi^2 L}{2EJ} & \frac{-\xi^3 L^2}{6EJ}\\ 0 & 1 & \frac{-\xi^2 L}{2EJ} & \frac{-\xi^2 L^2}{2EJ}\\ 0 & 0 & L & \xi L^2\\ 0 & 0 & 0 & L^2\end{bmatrix}$$ - matrix for the elastic system (9)

$$[\mathbf{F}_{res}(\xi)]=\begin{bmatrix}0 & 0 & -\sum_i r_{1i}(\xi-\zeta_i)h(\xi-\zeta_i) & -\sum_i r_{1i}\zeta_i(\xi-\zeta_i)h(\xi-\zeta_i)\\ 0 & 0 & -\sum_i r_{1i}h(\xi-\zeta_i) & -\sum_i r_{1i}\zeta_i h(\xi-\zeta_i)\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

matrix complementation by the non-elastic deformations (9a)

$$[\mathbf{u}'(\xi)]=\begin{bmatrix} v_1(\xi)+\sum_i\left(-r_{0i}+r_{1i}v_{1,\xi\xi}\right)\left(\zeta_i^-\right)\left(\xi-\zeta_i\right)h\left(\xi-\zeta_i\right) \\ \varphi_1(\xi)+\sum_i\left(-r_{0i}+r_{1i}v_{1,\xi\xi}\right)\left(\zeta_i^-\right)h\left(\xi-\zeta_i\right) \\ M_1(\xi) \\ T_1(\xi) \end{bmatrix} \tag{10}$$

The transfer matrix is derived from the substitution of the matrix values **[F$_{\mathbf{E}}$($\xi$)] ; [F$_{\mathbf{res}}$($\xi$)]** at the end of the bracket for $\xi = 1$:

$$[\mathbf{F}_E]=\begin{bmatrix} \frac{1}{L} & 1 & \frac{-L}{2EJ} & \frac{-L^2}{6EJ} \\ 0 & 1 & \frac{-L}{2EJ} & \frac{-L^2}{2EJ} \\ 0 & 0 & L & L^2 \\ 0 & 0 & 0 & L^2 \end{bmatrix}$$ - transfer matrix for the elastic system (11)

$$[\mathbf{R}]=\begin{bmatrix} 0 & 0 & -\sum_i r_{1i}\left(1-\zeta_i\right) & -\sum_i r_{1i}\zeta_i\left(1-\zeta_i\right) \\ 0 & 0 & -\sum_i r_{1i} & -\sum_i r_{1i}\zeta_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ matrix complementation of the transfer matrix by the non-elastic deformations (12)

If a beam has only one continuum bracket, four starting conditions can be calculated using the system of equations:

$$\mathbf{v_k} = \mathbf{F}\mathbf{v_0} \tag{13}$$

where: $\mathbf{v_0} = \{\mathrm{v}_0 \ \Phi_0 \ \mathrm{M}_0 \ \mathrm{T}_0\}$ - parameters at the beginning of the section;

$\mathbf{v_k} = \{\mathrm{v}_k \ \Phi_k \ \mathrm{M}_k \ \mathrm{T}_k\}$ - parameters at the end of the section;

**F** is a matrix which in general is a sum of the elastic bar solution effects and of the permanent bar deformation effects:

$$\mathbf{F} = \mathbf{F_E} + \mathbf{F_{res}} \tag{14}$$

### 3.2. Compressed element

The task can also be extended by the case in which, axial force is introduced into the structure (eg. prestressing reinforcement or a pillar). To the equation of the bent axis of the bar, a formula describing influence of the axial force is introduced:

$$\frac{\partial^4 v(\xi)}{\partial \xi^4}+\frac{Nl^2}{EJ}\frac{\partial^2 v(\xi)}{\partial \xi^2}=\frac{1}{EJ}\sum_i r_N\,\delta_{,\xi\xi}\left(\xi-\zeta_i\right) \tag{15}$$

The solution then has a form of recurrence formulae [2]:

$$v_n(\xi)=R_N v_{n-1}(\xi) \tag{16}$$

where: $R_N$ – recurrence factor of the equation (15).

The transfer matrix is derived from the substitution of the general differential equation:

$$v(\xi) = v(0) + \frac{1}{k} v,_{\xi}(0) + \left[\frac{1}{k^2}(1-\cos k\xi)\right] v,_{\xi\xi}(0)$$
$$+\left[\frac{1}{k^3}\left(\xi - \frac{\sin k\xi}{k}\right)\right] v,_{\xi\xi\xi}(0) \tag{17}$$
$$+\frac{1}{k}\sum_{i=1}^{n}\left[r_0 + r_1 v,_{\xi\xi}\left(\zeta_i^-\right)\right]\left[\sin k(\xi-\zeta_i)\right] h(\xi-\zeta_i)$$

where: $k = \sqrt{\frac{Nl^2}{EJ}}$ (18)

The unknown value $v,_{\xi\xi}(\zeta_i\text{-})$ is calculated from the recurrence formula:

$$v,_{\xi\xi}(z_i^-) = |A_i|\ v,_{\xi\xi}(0) + |B_i|\ v,_{\xi\xi\xi}(0) - |C_i| \quad , \tag{19}$$

the derivative of which is presented in detail in [2]. Matrices $A_i$ ; $B_i$ ; $C_i$ are formed by substituting the i column of matrix A with vectors {a}; {b}; {c}. Matrix A is a triangular matrix.

$\mathbf{A} = [a_{ij}]_{i=1,\ldots n}$
$\{a\} = (a_j)_{j=1,\ldots n}$
$\{b\} = (b_j)_{j=1,\ldots n}$
$\{c\} = (c_j)_{j=1,\ldots n}$

where:

$$a_{ij} = \begin{cases} 1 & \text{dla } i=j \\ 0 & \text{dla } i<j \\ kr_{1j}\sin k(\zeta_i - \zeta_j) & \text{dla } i>j \end{cases}$$

$$a_j = \cos k\zeta_j$$

$$b_j = \sin k\zeta_j$$

$$c_j = \begin{cases} 0 & i=1 \\ kr_{0i}\sin k(\zeta_i - \zeta_j) & i>1 \end{cases}$$

Hence, the following recurrence formulae can be derived for the matrices' determinants:

$$|A_i| = a_i - \sum_{j=1}^{i-1} |A_{i-j}|\, a_{i,i-j} \tag{20}$$

$$|B_i| = b_i - \sum_{j=1}^{i-1} |B_{i-j}|\, a_{i,i-j} \tag{21}$$

$$|C_i| = c_i - \sum_{j=1}^{i-1} |C_{i-j}|\, a_{i,i-j} \tag{22}$$

By means of the presented formulae, the algorithms for the numerical calculations can be formulated. The transfer matrix of the entire structure then will be:

$$v_k = \mathrm{C}_n\, H_{n-1}\, \mathrm{C}_{n-1} \mathrm{K}\, \mathrm{C}_2\, H_1\, \mathrm{C}_1\, v_0 = \mathrm{C}_{glob}\, v_0 \tag{23}$$

where: $\mathbf{C_i}$ – transfer matrix for the pillar (4.81):

b = l / l' ;

a = EJ / EJ'

l' ; EJ' - randomly selected comparative values

To solve the system of equations (23), known boundary conditions at both ends of the system are used. For a one-element articulated ends bar, the system of equations is:

$$C = \begin{bmatrix} 1 & \frac{\beta}{k} & -\frac{1}{\alpha}\left(\frac{\beta}{k}\right)^2\left[(1-\cos k)-\frac{1}{k}\sum_{i=1}^{n} r_{1i}|A_i|\sin k(1-\zeta_i)\right] & -\frac{1}{\alpha}\left(\frac{\beta}{k}\right)^3\left[\left(1-\frac{\sin k}{k}\right)-\frac{1}{k}\sum_{i=1}^{n} r_{1i}|B_i|\sin k(1-\zeta_i)\right] & \frac{1}{k}\sum_{i=1}^{n}\left(r_0-r_1|C_i|\right)\sin k(1-\zeta_i) \\ 0 & 1 & -\frac{1}{\alpha}\frac{\beta}{k}\left[\sin k-\frac{1}{k}\sum_{i=1}^{n} r_{1i}|A_i|\cos k(1-\zeta_i)\right] & -\frac{1}{\alpha}\left(\frac{\beta}{k}\right)^2\left[(1-\cos k)-\frac{1}{k}\sum_{i=1}^{n} r_{1i}|B_i|\cos k(1-\zeta_i)\right] & \sum_{i=1}^{n}\left(r_0-r_1|C_i|\right)\cos k(1-\zeta_i) \\ 0 & 0 & \left[\cos k-\frac{1}{k}\sum_{i=1}^{n} r_{1i}|A_i|\sin k(1-\zeta_i)\right] & \frac{\beta}{k}\left[\sin k-\frac{1}{k}\sum_{i=1}^{n} r_{1i}|B_i|\sin k(1-\zeta_i)\right] & \alpha k\sum_{i=1}^{n}\left(r_0-r_1|C_i|\right)\sin k(1-\zeta_i) \\ 0 & 0 & \frac{k}{\beta}\left[-\sin k-\frac{1}{k}\sum_{i=1}^{n} r_{1i}|A_i|\cos k(1-\zeta_i)\right] & \left[\cos k-\frac{1}{k}\sum_{i=1}^{n} r_{1i}|B_i|\cos k(1-\zeta_i)\right] & \alpha k^2\sum_{i=1}^{n}\left(r_0-r_1|C_i|\right)\cos k(1-\zeta_i) \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

$$c_{12}\, j_0 + c_{14}\, T_0 = c_{15} \tag{25}$$

$$c_{32}\, j_0 + c_{34}\, T_0 = c_{35}$$

The critical force informs about the instability of the structure. The state will correspond with the indeterminacy of the equation system, which can be expressed by condition:

$$\det\begin{bmatrix} c_{12}(k) & c_{14}(k) \\ c_{32}(k) & c_{34}(k) \end{bmatrix} = 0 \tag{26}$$

The equation has a lot of elements on critical values $k_n$ , from which values of critical foces $N_{kr}$ can be derived. For a one-element articulated boundary conditions bar, the age equation has a form:

$$\left(\frac{\beta}{k}\right)^2\left[\sin k-\frac{1}{k}\sum_{i=1}^{n} r_{1i}|B_i|\sin k(1-\zeta_i)\right]-\frac{1}{\alpha}\frac{1}{k}\sum_{i=1}^{n} r_{1i}|B_i|\sin k(1-\zeta_i)=0 \tag{27}$$

after transformation:

$$\beta^2 \sin k = \left(\frac{\beta^2}{k} + \frac{1}{\alpha}\right)\sum_{i=1}^{n} r_{1i}\left|B_i\right| \sin k\left(1-\zeta_i\right) \quad (28)$$

where: $|B_i|$ – defined by a recurrence formula.

### 3.3. A dynamically loaded element

Similar solution is observed in the task of determining the proper vibration of the bar and the generalized shifts and internal forces. They are obtained from a general differential equation describing vibration of the bar with the elastic and permanent dislocations [3]:

$$\frac{\partial^4 v(\xi,t)}{\partial \quad \xi^4} - \frac{ml^4}{EJ}\frac{\partial^2 v(\xi,t)}{\partial \quad t^2} = \sum_i r_K \qquad \delta_{,\xi\xi}(\xi - \zeta_i) \quad (29)$$

The solution is obtained in a form of recurrence formulae:

$$v_n(\xi,t) = R_K v_{n-1}(\xi,t) \quad (30)$$

where: $R_K$ – the recurrence factor of the equation (29).

For one element, the homogenous equation is:

$$\left.\frac{\partial^4 v(\xi,t)}{\partial \quad \xi^4} - \frac{ml^4}{EJ}\frac{\partial^2 v(\xi,t)}{\partial \quad t^2}\right|^e = 0 \quad (30)$$

For the set vibration it can be assumed that the solution has a form of a ratio (variables separation method). The solution has a form :

$$v^e(x,t) = T(t)\, V^e(x) \quad (31)$$

At the harmonic vibration:

$$T(t) = e^{iwt} \quad (32)$$

where: w – circular frequency of the harmonic vibration of the bar

The timeless equation can then have a form of:

$$\left. V_{,\xi\xi\xi\xi} - k^4 V\right|^e = 0 \quad (33)$$

$$k^4 = \omega^2 \frac{ml^2}{EJ} \quad (34)$$

The general integral of the differential equation (33) is:

$$V^e(\xi) = A \cos k\xi + B \sin k\xi + C \operatorname{ch} k\xi + D \operatorname{sh} k\xi \quad (35)$$

Based on the solution, it is possible to build a function which will also be a solution of the differential equation (33). The solution can be obtained using the shift method, the force method, or the mixed method. In the static task, the integral of a differential equation expressed by means of the initial parameters was presented. Now, a solution by means of the shift method is shown. Constant equations are then kinematic boundary conditions:

$$V^e(\xi) = d_i Z_1(k\xi) + j_i Z_2(k\xi) + d_j Z_3(k\xi) + j_j Z_4(k\xi) \quad (36)$$

where: $d_i = V^e(0)$

$j_i = V^e_{,x}(0)$

$d_j = V^e(1)$
$j_j = V^e_{,x}(1)$

The solution of the amplitude equation with their derivatives can be shown in a form of a matrix:

$$\mathbf{V^e(\xi) = Z(k\xi)\, d^e} \tag{37}$$

where: $\mathbf{V^e(\xi)} = \{V(\xi); V_{,x}(\xi); M(\xi); T(\xi)\}^e$
$\mathbf{d^e} = [\{ d_i ; j_i ; d_j ; j_j \}^e]^T$.

Matrix of the shape can be described by:

$$\mathbf{Z}(k\xi)=\begin{bmatrix} Z^1(k\xi) & Z^2(k\xi) & Z^3(k\xi) & Z^4(k\xi) \\ Z^1_{,\xi}(k\xi) & Z^2_{,\xi}(k\xi) & Z^3_{,\xi}(k\xi) & Z^4_{,\xi}(k\xi) \\ -EJZ^1_{,\xi\xi}(k\xi) & -EJZ^2_{,\xi\xi}(k\xi) & -EJZ^3_{,\xi\xi}(k\xi) & -EJZ^4_{,\xi\xi}(k\xi) \\ -EJZ^1_{,\xi\xi\xi}(k\xi) & -EJZ^2_{,\xi\xi\xi}(k\xi) & -EJZ^3_{,\xi\xi\xi}(k\xi) & -EJZ^4_{,\xi\xi\xi}(k\xi) \end{bmatrix} \tag{38}$$

where:

$$Z^1(k\xi)=S(k\xi)+\frac{K^2-SU}{U^2-KT}U(k\xi)+\frac{KU-ST}{KT-U^2}K(k\xi)$$

$$Z^2(k\xi)=\frac{1}{k}T(k\xi)+\frac{SK-TU}{U^2-KT}U(k\xi)+\frac{SU-T^2}{KT-U^2}K(k\xi)$$

$$Z^3(k\xi)=\frac{U}{U^2-KT}U(k\xi)+\frac{T}{KT-U^2}K(k\xi)$$

$$Z^4(k\xi)=-\frac{K}{U^2-KT}U(k\xi)+\frac{U}{KT-U^2}K(k\xi)$$

$$Z^1_{,\xi}(k\xi)=k\left[K(k\xi)+\frac{K^2-SU}{U^2-KT}T(k\xi)+\frac{KU-ST}{KT-U^2}U(k\xi)\right]$$

$$Z^2_{,\xi}(k\xi)=k\left[\frac{1}{k}S(k\xi)+\frac{SK-TU}{U^2-KT}T(k\xi)+\frac{SU-T^2}{KT-U^2}U(k\xi)\right]$$

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$$Z^4_{,\xi\xi\xi}(k\xi)=k^3\left[-\frac{K}{U^2-KT}K(k\xi)+\frac{U}{KT-U^2}S(k\xi)\right] \tag{39}$$

whereas Krylow's functions:

$$\begin{aligned} K(k\xi)&=0{,}5(\mathrm{sh}k\xi-\sin k\xi), \quad K=K(k)\\ U(k\xi)&=0{,}5(\mathrm{ch}k\xi-\cos k\xi) \quad U=U(k)\\ T(k\xi)&=0{,}5(\mathrm{sh}\,k\xi+\sin k\xi), \quad T=T(k)\\ S(k\xi)&=0{,}5(\mathrm{ch}k\xi+\cos k\xi), \quad S=S(k) \end{aligned} \tag{40}$$

are in a form of tables and have the following values:

$K(0) = U(0) = T(0) = 0;\ S(0) = 1;$ (41)

$K_{,xxxx}(kx) = k\, K_{,xxx}(kx) = k^2\, T_{,xx}(kx) = k^3\, S_{,x}(kx) = k^4\, K(kx)$

The transfer matrix in the static task enables the determination of deformations and forces at the end of the bracket by means of the initial parameters, whereas here the static values in any point of the bracket are expressed by means of deformations at its ends. In order to maintain a uniform solution it is necessary to determine the generalized deformations and forces at the ends of the bracket and find a mutual relation between them:

$$\mathbf{V^e(0) = Z(0)}\ \boldsymbol{\delta^e} \qquad (42)$$

$$\mathbf{V^e(1) = Z(k)}\ \boldsymbol{\delta^e} \qquad (43)$$

Determining vector $\delta^e$ from fomula (42) and substituting it in equation (43) one obtains:

$$\boldsymbol{\delta^e} = \mathbf{Z^{-1}(0)\ V^e(0)} \qquad (44)$$

$$\mathbf{V^e(1) = Z(k)\ Z^{-1}(0)\ V^e(0)} \qquad (45)$$

The transfer matrix is then:

$$\mathbf{F} = \mathbf{Z(k)\ Z^{-1}(0)} \qquad (46)$$

$$\mathbf{F} = \begin{bmatrix} S & \frac{\beta}{k}T & -\frac{1}{\alpha}\left(\frac{\beta}{k}\right)^2 U & -\frac{1}{\alpha}\left(\frac{\beta}{k}\right)^3 V \\ \frac{k}{\beta} & S & -\frac{1}{\alpha}\frac{\beta}{k}T & -\frac{1}{\alpha}\left(\frac{\beta}{k}\right)^2 U \\ -\alpha\left(\frac{k}{\beta}\right)^2 U & -\alpha\frac{k}{\beta}V & S & \frac{\beta}{k}T \\ -\alpha\left(\frac{k}{\beta}\right)^3 T & -\alpha\left(\frac{k}{\beta}\right)^2 U & \frac{k}{\beta}V & S \end{bmatrix} \qquad (47)$$

where: a = $EJ^e / EJ'$ ;

b = $l^e / l'$ ;

EJ' ; l' - comparative rigidity and length

## 4. SUMMARY

Presented method allows to analyse the construction based on the solution of the elementar differential equation

$$G_{,xxxx} = \delta(\xi - \zeta)$$

If the function G (ξ - ζ) will be multiplied (inside the integral) by the right side of the differential equation, than we obtain a solution that takes into account the impact of the permanent and rheologycal deformations.

$$v_1(\xi) = \int_0^1 G(\xi,\zeta)\, p^O(\xi)\, d\xi$$

where: $p^O(\xi)$ – load and impact of the permanent and rheologycal deformations.

This presentation of problem allows in a wider aspect to use the solutions from the classical theory of elasticity for materials with variable characteristics.

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