

## THE APPLICATION OF POD CURVES TO DAMAGE DETECTION BASED ON PARTIAL MODELS- A NUMERICAL AND EXPERIMENTAL STUDY

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**Abstract.** *Non-destructive techniques for damage detection became the focus of engineering interests in the last few years. However, applying these techniques to large complex structures like civil engineering buildings still has some limitations since these types of structures are unique and the methodologies often need a large number of specimens for reliable results. For this reason, cost and time can greatly influence the final results.*

*Model- Assisted Probability Of Detection (MAPOD) has taken its place among the ranks of damage identification techniques, especially with advances in computer capacity and modeling tools. Nevertheless, the essential condition for a successful MAPOD is having a reliable model in advance. This condition is opening the door for model assessment and model quality problems.*

*In this work, an approach is proposed that uses Partial Models (PM) to compute the Probability Of damage Detection (POD). A simply supported beam, that can be structurally modified and tested under laboratory conditions, is taken as an example. The study includes both experimental and numerical investigations, the application of vibration-based damage detection approaches and a comparison of the results obtained based on tests and simulations.*

*Eventually, a proposal for a methodology to assess the reliability and the robustness of the models is given.*

## 1 INTRODUCTION

The main objective of damage identification is to detect the damage in the early stages, so as to reduce the risks of stability failure and serviceability issues. Since civil engineering structures possess characteristics that make them different than systems in other engineering fields, this goal is not easily attained. In most cases, given that each structure is unique, the results from a certain method applied to one structure are not valid for other structures. These structures also cannot be moved or isolated in order to perform ideal damage detection tests, such as those conducted in the laboratory. In addition, the detection tests should be conducted while the structure is operational, making some areas of the structure inaccessible, and precluding anything other than non-destructive testing.

Although simulation tools have developed significantly in the last few years, the quality of models remains a serious problem that faces engineers. This is due to of different sources of uncertainty. In general, uncertainty can be classified as either: aleatoric or epistemic. The former one represents the randomness of the phenomenon and the second is related to lack of the knowledge and the data. This categorization helps deal with uncertainties in the proper way e.g. to reduce the epistemic uncertainty, further and deeper studies or more data collection is needed. However, the aleatoric uncertainty can be included in the model stochastically.

In the case of damage identification, uncertainty included in specimens and uncertainty in experiments are the two main components of total uncertainty. Uncertainties related to specimens include dimensions, supporting conditions and loading conditions. Furthermore, noise because of sensors can be addressed as an uncertainty included in experiments.

In most cases, it is neither easy nor practical to give an absolute answer for appearance of damage in a structure. This is due to uncertainties. Furthermore, trying to eliminate the noise completely requires much time and efforts. Therefore, it is very important to define a minimum level of confidence which can be the balance between sufficient accuracy and required effort. Choosing a reliable confidence level can be done based on the acceptable risk criteria.

An overview of previous research, which focused on using statistical information for damage assessment in civil engineering structures using vibrational based inspection can be found in [1]. However, only since 2004 has the MAPOD working group started to develop strategies to couple the physics-based and empirical understanding guided by draft protocols. This group was formed by the Air Force Research Laboratory, the FAA Technical Center and NASA. The motivations of a Model-Assisted Probability of Detection (MAPOD) approach are presented in [2]. It was mentioned that a wave of new inspection requirements is anticipated in the coming years to reduce the cost and time spent. The MAPOD approach was defined as an alternate approach which is sorely needed to reduce the amount of empirical tests. This approach depends on physics-based models to determine the POD. The POD is based on distributions of signal (from flaw of the same nominal size) and noise. These distributions are controlled by several factors that can be predicted by simulation tools such as MAPOD. However, empirical tests are still needed because many variabilities can not be described by well-understood physical phenomena. It is mentioned that the major goal of current activities is to codify methods that are less time/cost intensive than in MIL-HDBK-1823 (Nondestructive Evaluation System Reliability Assessment). The ideas and strategies to develop the MAPOD approach are introduced. In [3] the recent work and development as well as future work of the MAPOD working group are presented. In [4] a modified MAPOD approach is used to detect the damage in the case of an two-layer airframe structure. In [5] the MAPOD approach is applied to validate the reliability of an automated ultrasonic inspection used for crack detection at fastener holes in the lower wing

skins of F-111 aircraft. A transfer function method is used to predict the POD for an angle-beam ultrasonic inspection of cracks in fastener holes within a complex structure. The MAPOD for ultrasonic structural health monitoring SHM is discussed in [6] as an effective technique for monitoring fatigue-induced damage. The paper introduces the differences between the traditional POD approaches as used for non-destructive evaluation and for SHM. In [7] a new Model-Assisted probabilistic reliability assessment methodology is described. The feasibility of applying this approach to typical sensing methods found in SHM systems is discussed. The POD is used in [8] to assess vibration-based damage identification techniques using different types of indicators, which depend, in general, on mode shapes. In [9], a POD model is used with a reliability-based crack growth model to assess fatigue damage in bridges.

In this work, a simply supported beam, that can be modified and tested under laboratory conditions, is studied including both experimental and numerical investigations, (figure 1). It has a length of 3300 mm and a cross section of type IPE 80 and typical steel material properties. The beam is simply supported at 50 mm from each end. In order to perform non-destructive tests, the beam is backed up with plates of size  $150 \times 46 \times 2.5 \text{ mm}^3$ . The damage is simulated by removing one or more of these plates. Three damage cases are studied: case 1: one plate is removed near the middle of the beam; case 2: a plate is removed near the support of the beam; case 3 two plates are removed near the middle and a support of the beam. The work uses vibration-based method for damage detection. 16 accelerometers are used to record acceleration at the top of the beam.

## 2 PARTIAL MODELS

### 2.1 Structure Model

There are different possibilities to model the studied beam. It can be modeled using beam, shell or solid elements. This is dependent on the uncertainty types that one wants to include in these models. For example, in reality, supports are at the bottom of the cross section; however, in the case of using beam elements to simulate the beam, the supports are at the center of the cross section. Therefore, to include this uncertainty, a more sophisticated model is needed,(figure 2).

### 2.2 Damage Model

The types of damage that structures can suffer vary depending on different conditions e.g. the material, operation task, quality of construction, the effect of the surrounding environment, etc. For instance, in concrete structures damage can appear as a crack which can be harmful or harmless according to its position, type, propagation opportunity and also the type of structure itself which will be the final judge of the accepted tolerance e.g. crack tolerance for dams is more conservative than for normal buildings.

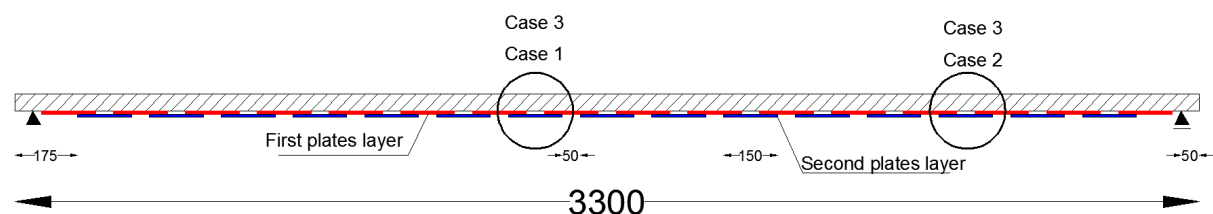


Figure 1: The studied beam sketch showing some dimensions and the distribution of the plates. Dimensions in mm

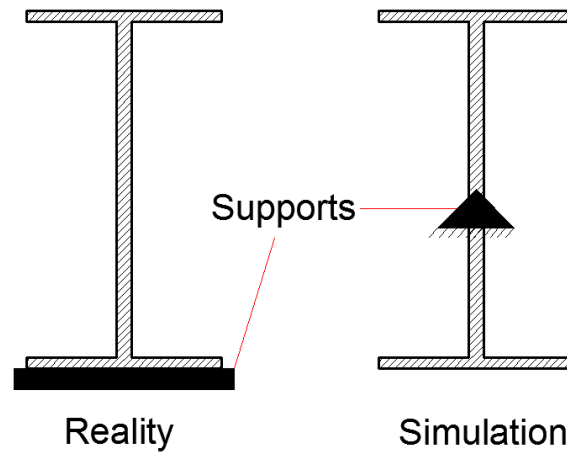


Figure 2: A cross section of the studied beam at place of the support, uncertainty because of support place in case of using beam elements to model the beam

Non-destructive techniques only allow non-destructive damage. In this example, the non-destructive damage will be represented by the removal of one or more plates, (figure 1). However, this damage model is not the real damage that should be detected in the structure.

Nevertheless, if the global model can be validated for different types of non-destructive damage at different locations with differing severities, this will produce a strong argument that the real damage would be represented correctly using this model. In this work, two types of real damage are used: decreasing the thickness of the bottom flange of the beam and E modulus degradation.

### 2.3 Indicator

In general, the data which is recorded from a damaged structure contains information about damage as well as noise from different sources of uncertainty. In order to extract this damage information, appropriate indicators are needed. In general, indicators are categorized according to the level of damage information that they can provide as following,[1]:

- 1- Level1: identification of damage;
- 2- Level2: level1 and the location of damage;
- 3- Level3: level2 and severity of damage;
- 4- Level4: level3 and prediction of the remaining service life.

The computational method of POD value is dependent on the level of information that indicators can provide. For instance, if the indicator shows damage in the correct location, it has truly detected damage. Conversely, if the indicator shows damage, but this damage is in the false location, it is only a false alarm and this damage has not been detected on the existing structure.

In general, two types of indicators can be used in the case of the vibration-based method. The first depends on the mode shapes and their derived features. The second is based on analyzing the signal directly e.g. stochastic subspace identification method (SSI). In this work, only the first type will be presented.

### 2.3.1 Indicators Based on Mode Shapes

This group of indicators utilizes post-processed data. In other words, the mode shapes should be calculated either by modal analysis or records analysis. After that, the mode shapes are processed to find if it is possible to distinguish between damage and noise. Three indicators are used: mode shapes, mode shape curvatures and model strain energy. The mode shape is calculated using equation ( 1)

$$Ind_j = (\phi_{0i}^2 - \phi_{ji}^2)^2 \quad (1)$$

where:

$Ind$ : the indicator's value

$j$ : the damage case

$\phi_{0i}$ : the mode shape  $i$  of non damaged beam

$\phi_{ji}$ : the mode shape  $i$  of the damaged beam case  $j$

Mode shapes curvatures are calculated approximately by finding the second derivative of the mode shapes with respect to position. The model strain energy indicator is derived from the Euler-Bernoulli beam of length  $l$  as shown in equation ( 2)

$$U_i = \frac{1}{2} \int_0^l (EI) \left( \frac{d^2 \phi_i}{dx^2} \right)^2 dx \quad (2)$$

where:

$U$ : the strain energy  $i$ : index of the mode shape

$EI$ : the structure rigidity

$\phi_i$  is the mode shape number  $i$

Other indicators can be found in the literature, such as Yuen functions, Dynamic Flexibility, etc...

### 2.3.2 Indicators Based Signal Analysis

This group of indicators utilizes pre-processed data. Stochastic subspace identification method (SSI) is an example for this type of indicator. The main advantages of this type of indicator are that they are more sensitive and present less data analysis uncertainty. However, the level of information that they provide is low.

## 2.4 Sensor Position

Since the indicators in this example depend on the quality of the mode shapes, mode shapes errors can be used as an objective function for the optimization process. The number of sensors used in these experiments is limited by availability in the laboratory and therefore 16 accelerometers are used. Since there are a limited number of accelerometers, the optimization process involves finding the best 16 nodes that will lead to a minimum value of the objective function,  $f$ , as represented by equation ( 3).

$$f = \|\phi_{133} - \phi_{16}\| \quad (3)$$

where:

$\phi_{133}$ : the mode shapes generated using the total number of nodes of the model

$\phi_{16}$ : the mode shapes generated using only 16 nodes

## 2.5 Excitation

An excitation partial model is used when the experiments are simulated. In this work, only impulse excitation is used. Since a force sensor is available in the lab, it was possible to determine the range of the force value and its duration.

## 2.6 Threshold

Another essential element in calculating POD value is threshold level. Generally, when the indicator value crosses the threshold, this is a sign the damage exists. This value should be chosen carefully since the POD will be calculated based on its value. In this work, threshold is taken as a percentage of the maximum value of the indicator. The following levels are chosen: 70%, 75%, 80%, 85%, 90% and 95%. Furthermore, threshold plays an important role in evaluating the robustness of the global model, as will be shown later.

# 3 PRINCIPLES OF SELECTION AND ASSESSMENT METHODOLOGY

Since a large number of partial models can be developed as shown above, the number of combinations or global models will be much larger. As a result, it is impractical to apply the assessment process to each global model, especially if the complexity increases meaning more computational time and higher computer efforts are required. Therefore, it is necessary to develop a strategy to selectively limit the number of models in order to increase the efficiency of model quality assessment.

## 3.1 Model Selection

In the case of damage identification, the number of partial models can be minimized based on POD value decomposition. The decomposition process can be completed by classifying the uncertainty's sources into different independent types. The idea behind the decomposition is that some sources of uncertainty are easy to estimate and to apply and others are too complicated. As a result, checking models for each independent type of uncertainty helps to reduce the amount of models each time. In the decomposition stage, the quality of the models is not assessed but rather the quality of coupling.

In this example, the calculation of POD is decomposed into two independent types. The first is caused by the uncertainty between the samples or specimens,  $POD_s$ , and the second is caused by the uncertainty included in experiments,  $POD_e$ .  $POD_s$  is calculated as the ratio between the number of samples that show damage and the total number of samples.  $POD_e$  is calculated as the ratio between the number of the experiments that show damage and the total number of experiments. The final POD value is computed by multiplying both POD components together.

### 3.2 Model Assessment

In general, reliability and robustness are used to assess the quality of studied global models. However, in case of no existing damage, the model should show low probability of detection otherwise the model is unreliable. Furthermore, the model is considered reliable if damage is detected early with a high probability of detection.

The global model is considered robust if the following conditions are satisfied: the POD increases when damage level increases and changing an input parameter by a small amount does not lead to failure or unacceptable variation of the outcomes.

## 4 NUMERICAL RESULTS

In the following sections, only a few results are presented in order to show the application of the methodology that is given in the previous section. Since uncertainty between samples is easier to estimate and  $POD_s$  is more quickly calculated in this case, all possible global models that can be created by coupling partial models are checked for  $POD_s$ . Consequently, only few coupled partial models or global models are needed to be checked for the  $POD_e$  and finally assess their quality based on POD curves.

### 4.1 $POD_s$ of Non-destructive Damage

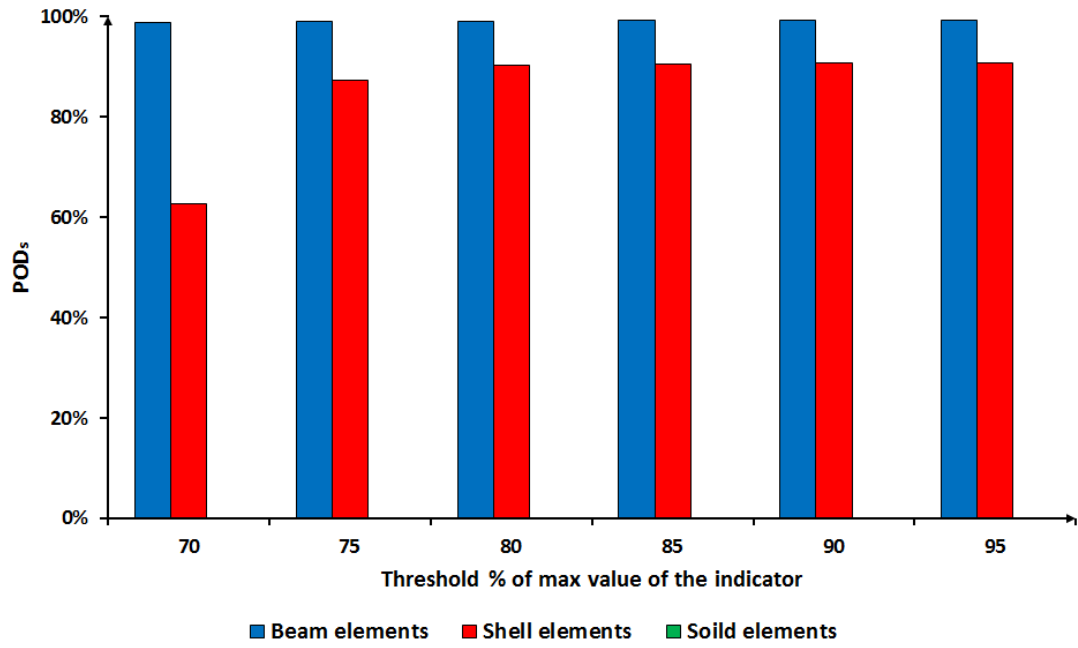
Figure( 3) shows  $POD_s$  results of two global models. The vertical axis represents  $POD_s$  and the horizontal axis represents threshold level. Threshold level is computed as a percentage of the maximum value that is given by the indicator. Damage is detected when the value of the indicator coincides with damage over the threshold level at the damage location. At other locations, the indicator's value is under this threshold level. The colors represent the type of beam partial model which is used in this global model. Blue represents the beam elements model, red represents the shell elements model and green represents the solid elements model. The results show that the solid elements model is not suitable for this model combination since the  $POD_s$  value did not cross 50% in the best case. In addition, for the low threshold level, coupled partial models with the shell elements model of the beam had low  $POD_s$  values, which means that this global model is not robust. However, the beam elements model correlated best with the other partial models.

Although some coupled partial models produced poor results, the quality of the these partial models is not necessarily a factor. It just means that the combination(s) used resulted in poor data. New partial models could be developed to produce better results or one could select only those models that produced the best results, eliminating all others.

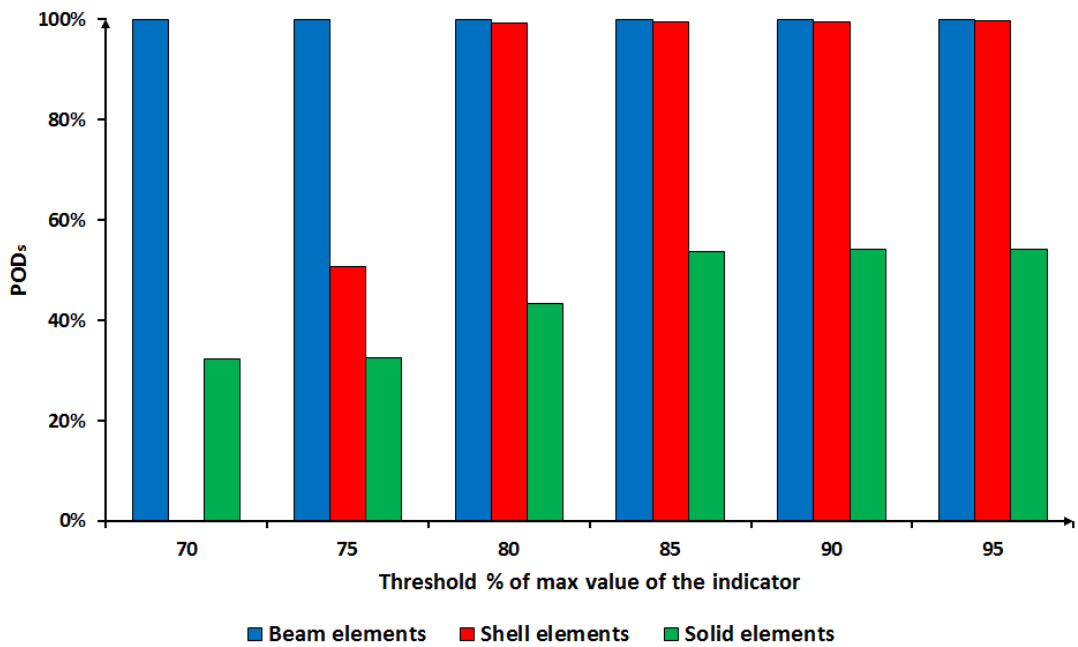
### 4.2 $POD_e$ of Non-destructive Damage

Although it is not easy to determine all sources of uncertainty in the experiments and the  $POD_e$  requires more time to calculate,  $POD_s$  results minimized the amount of partial model combinations that need to be checked. Only the beam elements model, strain energy and mode shape curvature will be considered in this part. Damage model and sensor positions remain the same as in the first part.

In order to excite a large number of mode shapes as much as possible, the beam is excited about  $0.15L$  from one of its supports. Excitation location remains the same for all tests. A high frequency rate is used to capture high order mode shapes. The signal is computed where the



(a)  $POD_s$ , damage case 1, mode shapes curvature indicator



(b)  $POD_s$ , damage case 1, strain energy indicator

Figure 3:  $POD_s$  value, damage case 1



sensors are supposed to be placed before the addition of noise to it. The noise is considered white Gaussian noise. The amplitude of the noise is estimated directly from the sensors.

One of model reliability conditions is that the POD value should be low in places where no damage exists. Therefore, the  $POD_e$  is calculated at each point where the sensors supposed to be placed. Figure( 4) shows the results for coupled partial models using the strain energy indicator for both damage cases 1 and 2. The vertical axis represents  $POD_e$  and the horizontal axis represents the length of the beam. Red triangles represent the position of the sensors. It shows a high  $POD_e$  value where damage exists. However, in places where no damage exists,  $POD_e$  can reach 50%. As a result, the reliability of the selected global model is low. The reasons could be that using an inappropriate noise model or uncertainty in the mode shape extraction and data analysis affected the results. In addition, the number of experiments can influence the results.

Figure( 5) shows the  $POD_e$  for both damage case 1 and 2 using the strain energy indicator. The vertical axis represents  $POD_e$  and the horizontal axis represents the threshold level. The results show that the selected global models are robust where damage exists because the  $POD_e$  value does not change significantly by changing the threshold level.

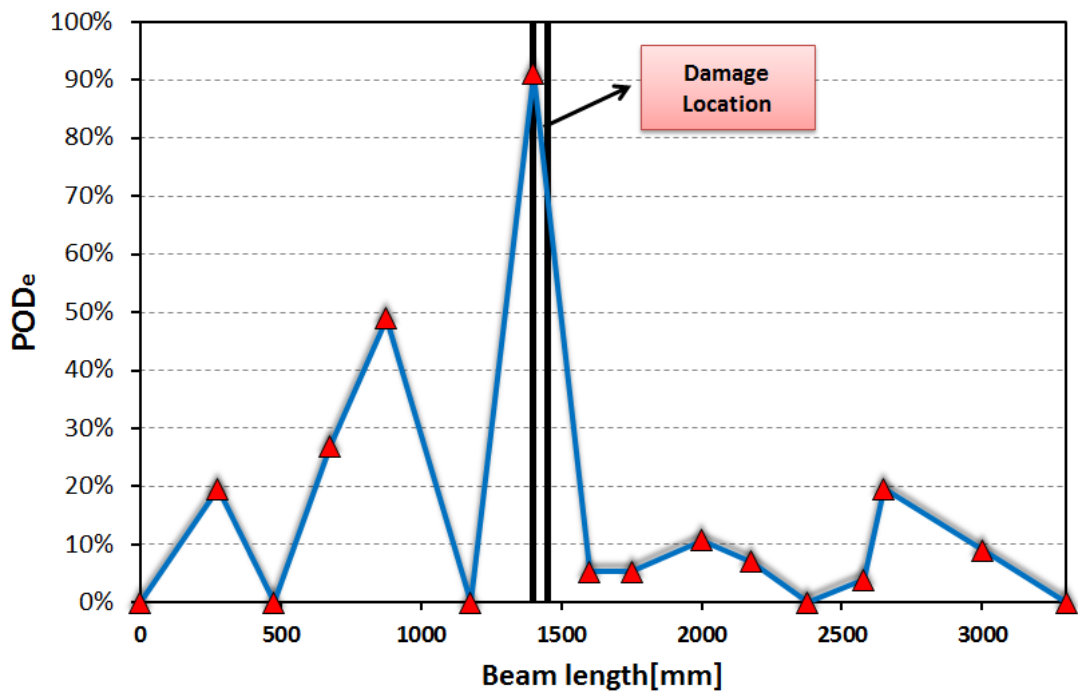
### 4.3 $POD_s$ Curves

As mentioned before, if the global model can be validated for different types of non-destructive damage in different locations with differing severities, this will result in a strong argument that the real damage would be represented correctly using this model. Furthermore, in order to assess the global models, POD curves should be created by replacing non-destructive damage with real damage detected in a structure. In the case of the E modulus degradation damage model, the damage increases from 0% to 20% with an interval of 1%. In the case of decreasing the thickness of the bottom flange, the damage increases from 0% to 80% with an interval of 4%. The  $POD_s$  is calculated in each step. The same procedure can be followed when creating  $POD_e$  curves.

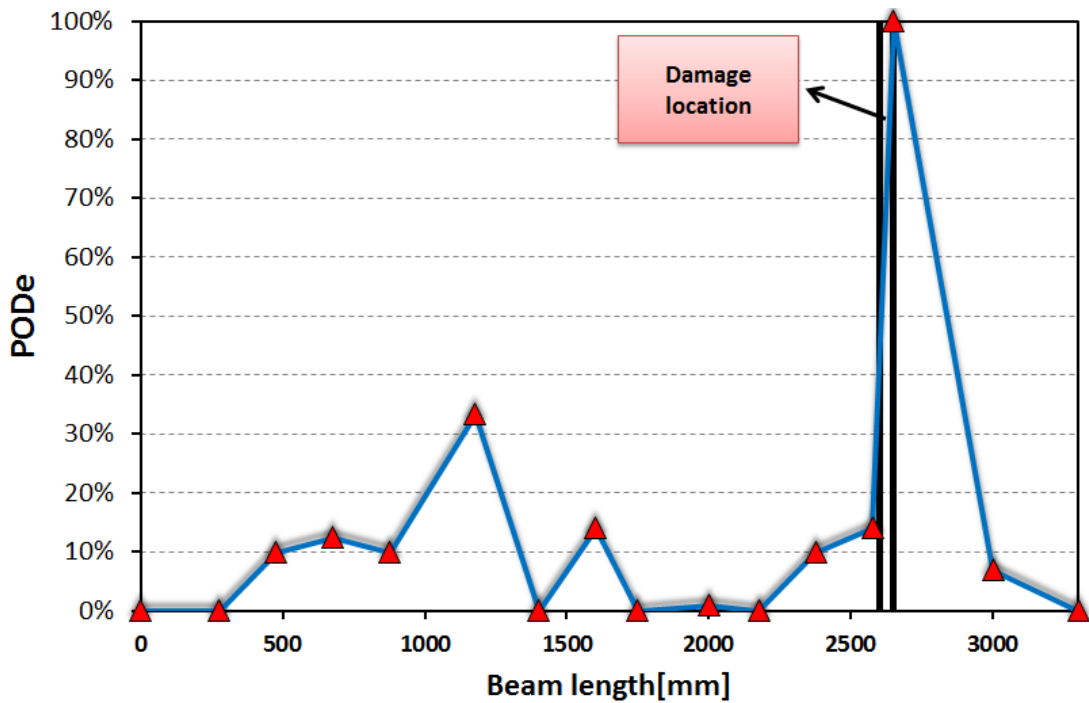
After producing the POD curves, model assessment principles can be applied again to evaluate the final global models not only for a certain damage level, but for the whole damage range. In figure ( 6) (a), if the E modulus degradation damage model is used, the global model is not robust since changing the threshold value leads to large variation in  $POD_s$  values e.g. if E modulus decreased about 12.5%, the  $POD_s$  value varies about 60% in the case of different threshold levels. The robustness of the model is improved if the model where damage is simulated by decreasing the thickness of bottom flange is used, figure ( 6) (b).

In figure ( 7) (b), if mode shape curvature is used, the global model is not robust since the  $POD_s$  value decreases by increasing the damage level. However, using the strain energy indicator, instead of mode shape curvature, improved the robustness of the model, figure ( 7) (a).

Model reliability can be estimated by the level of the damage that can be detected with high probability. Consequently, the model with results shown in figure ( 6) (b) is more reliable than the one with results shown in figure ( 7) (a).

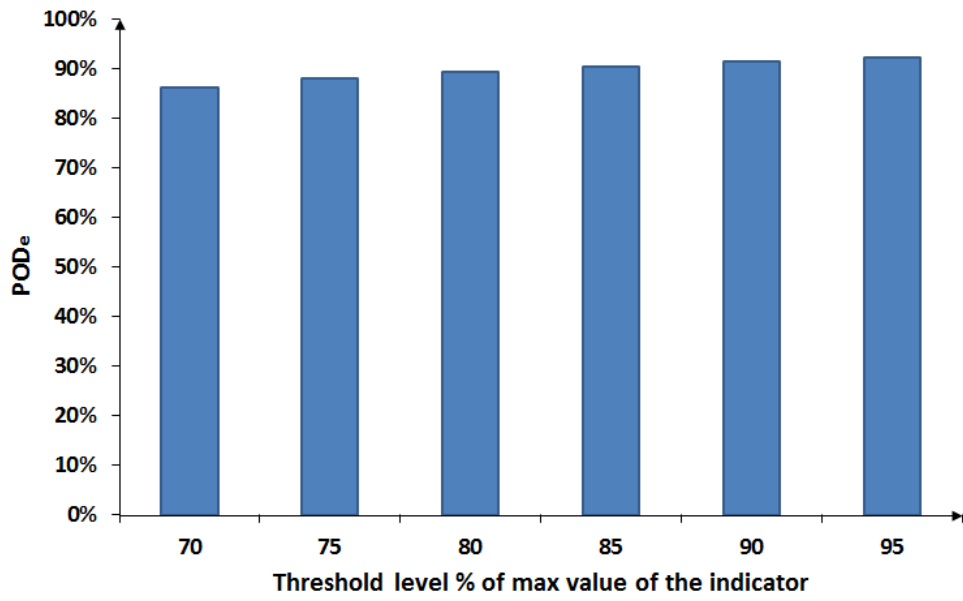


(a)  $POD_e$  in damage case 1, simulated experiments, strain energy indicator

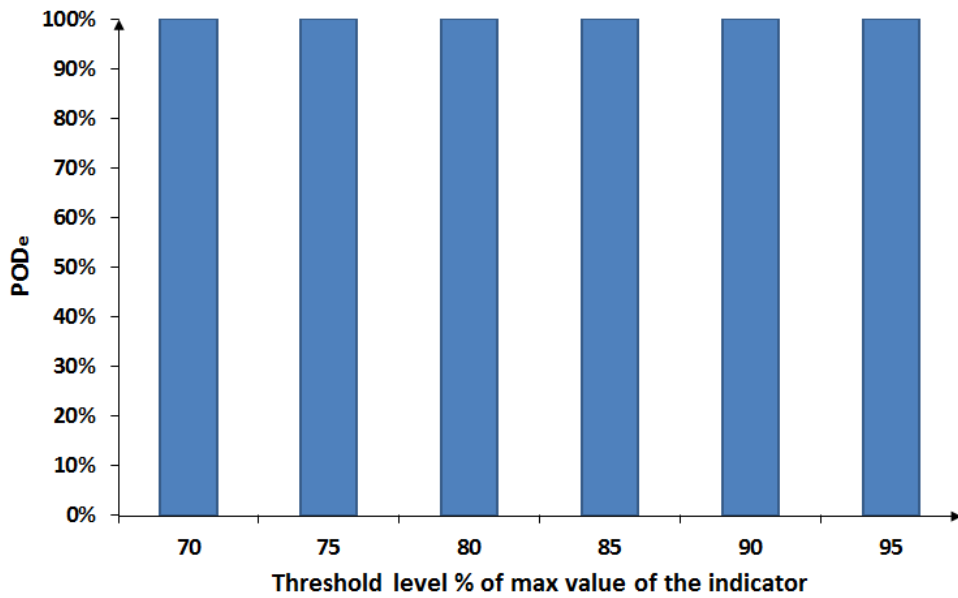


(b)  $POD_e$  in damage case 2, simulated experiments, strain energy indicator

Figure 4: Checking model reliability

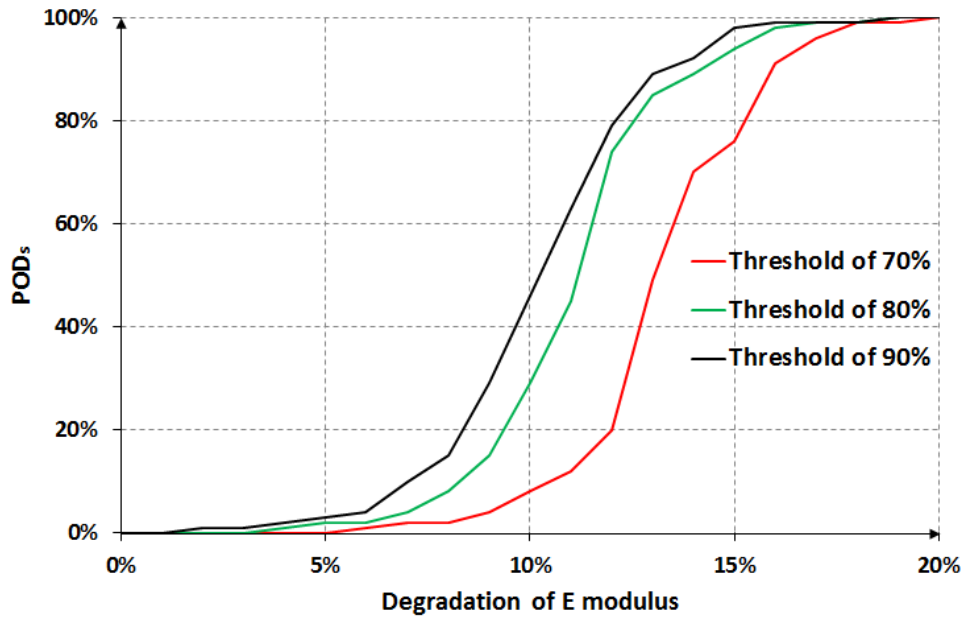


(a)  $POD_e$ , damage case 1, strain energy indicator

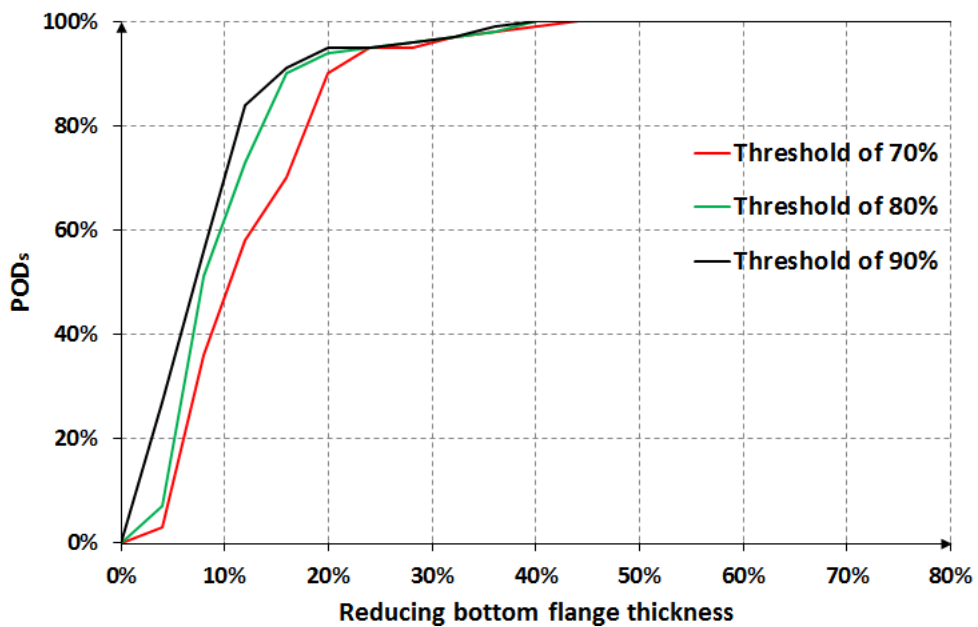


(b)  $POD_e$ , damage case 2, strain energy indicator

Figure 5:  $POD_e$  value, damage case 1

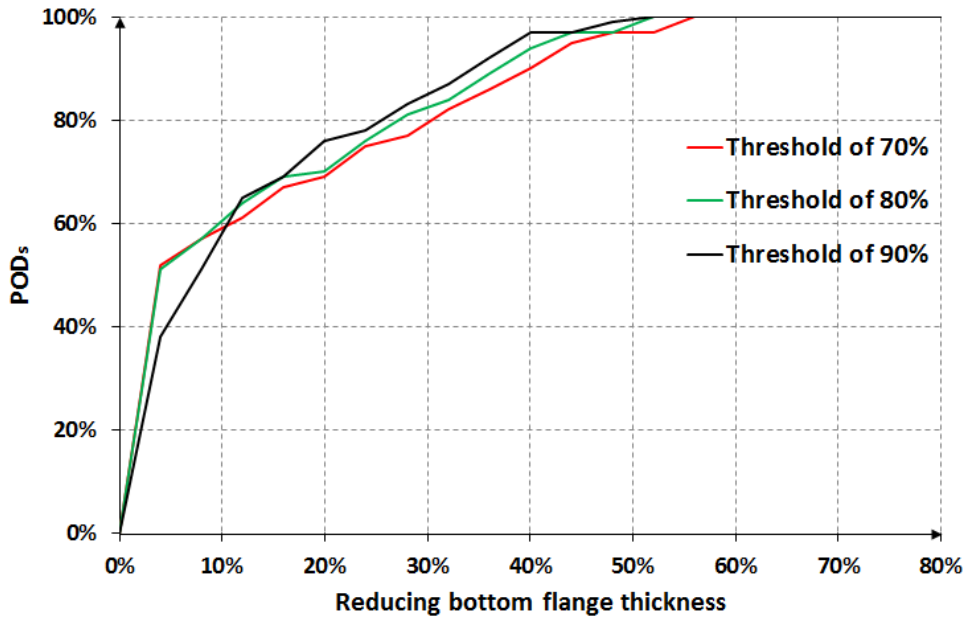


(a)  $POD_s$  curves, damage case 2, mode shapes curvature indicator, beam elements model, E modulus degradation model

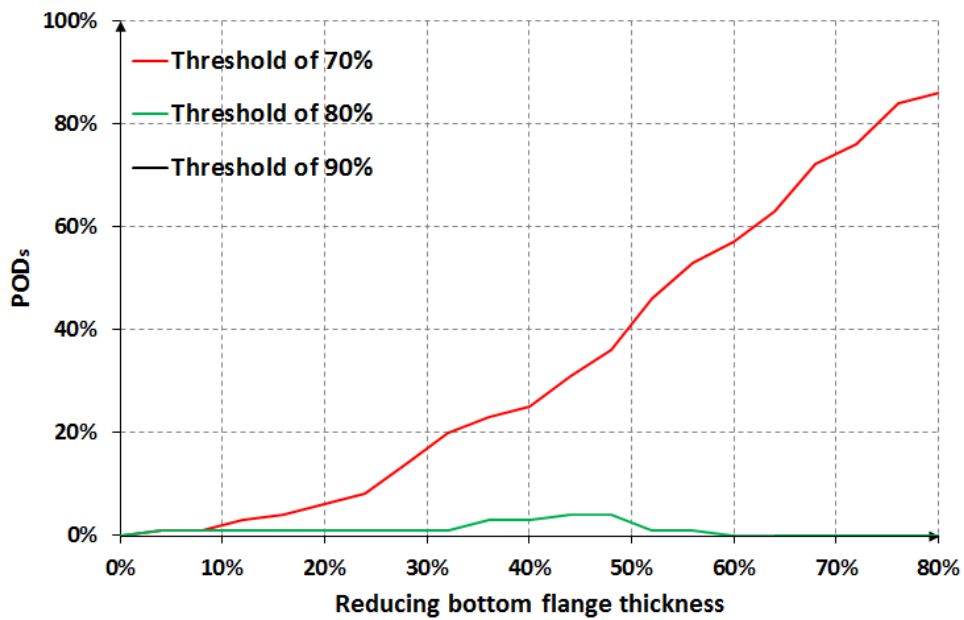


(b)  $POD_s$  curves, damage case 2, mode shapes curvature indicator, beam elements model, decrease the thickness of the bottom flange damage model

Figure 6:  $POD_s$  curves, Damage case 2



(a)  $POD_s$  curves, damage case 3, strain energy indicator, beam elements model, decrease the thickness of the bottom flange damage model



(b)  $POD_s$  curves, damage case 3, mode shapes curvature indicator, beam elements model, decrease the thickness of the bottom flange damage model

Figure 7:  $POD_s$  curves, Damage case 3

## 5 EXPERIMENTAL RESULTS

Experiments are performed based on numerical results using 16 accelerometers. A high frequency rate is used to capture high order modes. An impulse force is applied about  $0.15L$  from one of the beam supports to excite a large number of mode shapes. The excitation point remains the same in all tests. The damage of the first and second cases is produced by removing one plate in the damage area. In the third case, two plates are removed. In order to keep the mass constant, the plates are placed again on the beam in such way that they do not contribute to the stiffness anymore.

The recorded data is analyzed using the Stochastic Subspace Identification method (SSI) to extract the dynamic properties of the beam. After that, the indicators are applied to mode shapes to calculate the  $POD_e$ .

### 5.1 Global Physical Model Quality

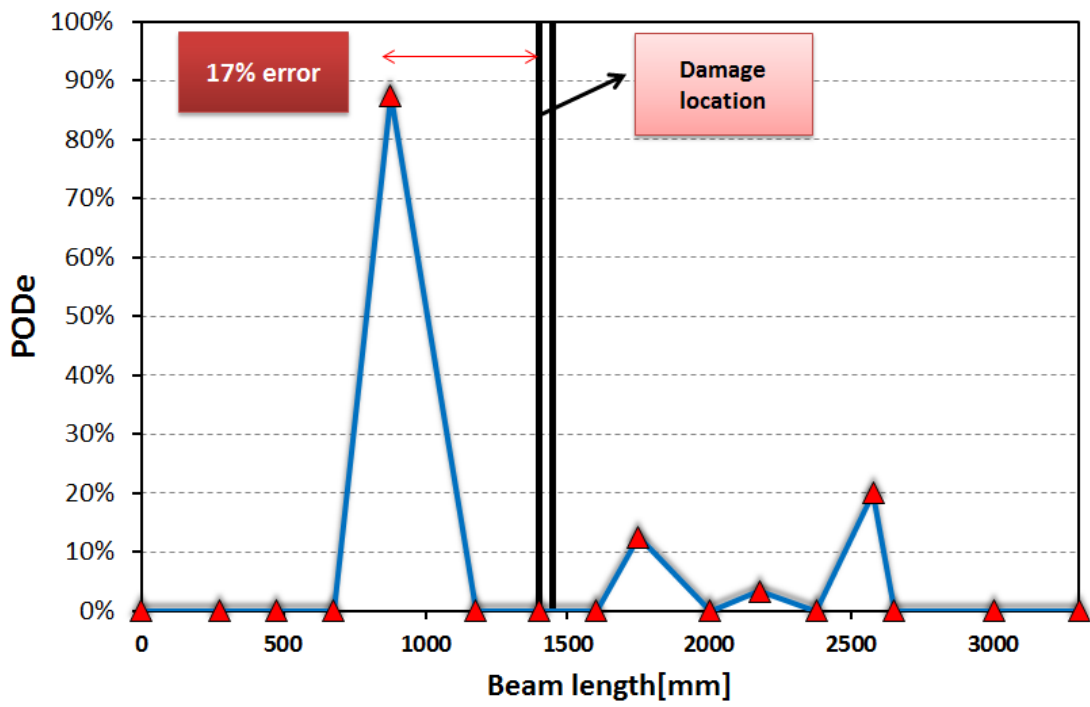
In the case of real experiments, as well as in simulated experiments, the reliability of the model should be checked. Therefore, the  $POD_e$  is calculated in each point where the sensors are placed. Figure( 8) shows the results for coupled physical partial models using the strain energy indicator for both damage cases 1 and 2. The Vertical axis represents  $POD_e$  and the horizontal axis represents the length of the beam. Red triangles represent the position of the sensors. It shows that in damage case 1, the  $POD_e$  is as high as in simulated experiments but damage location is differs by more than half meter, or 17% of the length of the beam, to the left. In damage case 2, the damage location is correct but the  $POD_e$  is not more than 70%, while in simulated experiments it is 100%. However, in places where no damage exists, the  $POD_e$  can reach 30%. The reason could be data analysis uncertainty. In addition, the number of experiments can influence the results.

Figure( 9) shows the  $POD_e$  for both damage case 1 and 2 using the strain energy indicator. The vertical axis represents  $POD_e$  and the horizontal axis represents threshold level. The results show that the selected global model is robust for damage case 1 but it is not for damage case 2, since the  $POD_e$  value changes significantly when the threshold from 70% to 80%.

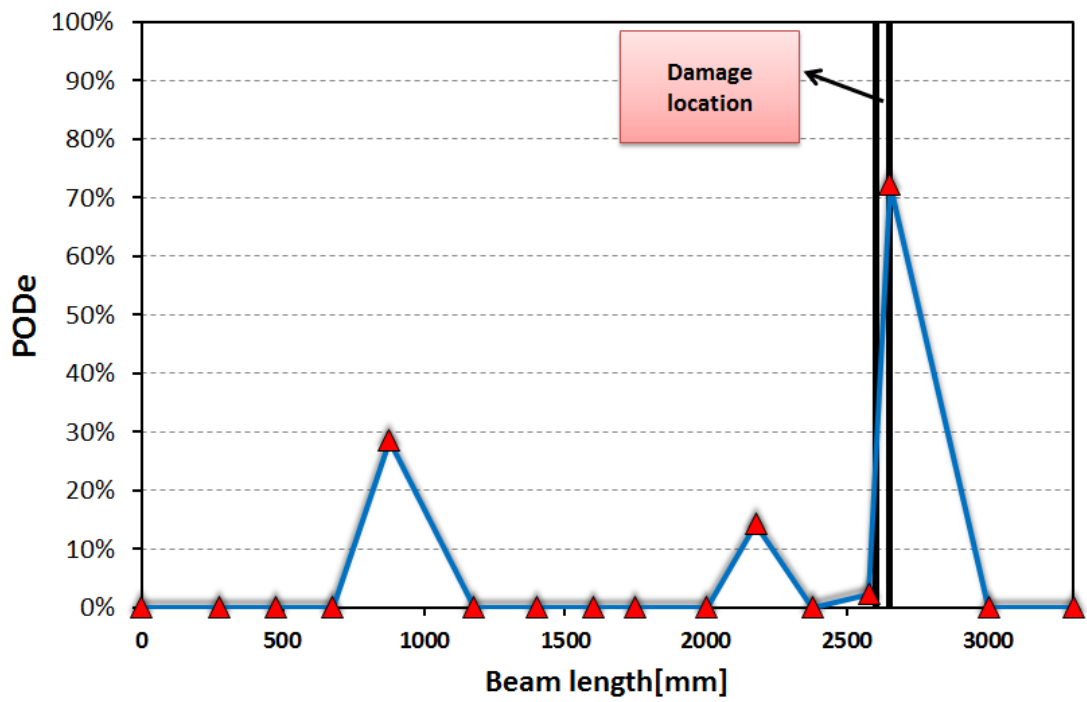
## 6 CONCLUSIONS

The application of POD curves to damage detection based on partial models is discussed. The work proposed an approach to calculate the **Probability Of damage Detection (POD)** based on **Partial Models (PM)**. However, the quality of the model is a problem that should be investigated if a Model assisted tool is used. Therefore, the work proposed a methodology to select and assess coupled partial models. The methodology is applied to a simply supported beam that can be modified and tested under laboratory conditions. Non-destructive damage is produced by removing one or more plates based on studied damage cases. The work uses a vibration-based method for damage detection. 16 accelerometers are used to record acceleration at the top of the beam.

The results show that the efficiency of model selection and assessment is increased when the POD is decomposed into two independent types:  $POD_s$  and  $POD_e$ , since the number of models is minimized. After the validation process, the POD curves can be created by replacing non-destructive damage with real damage that should be detected. In this work, two types of damage are used: E modulus degradation and reduction of bottom flange thickness.

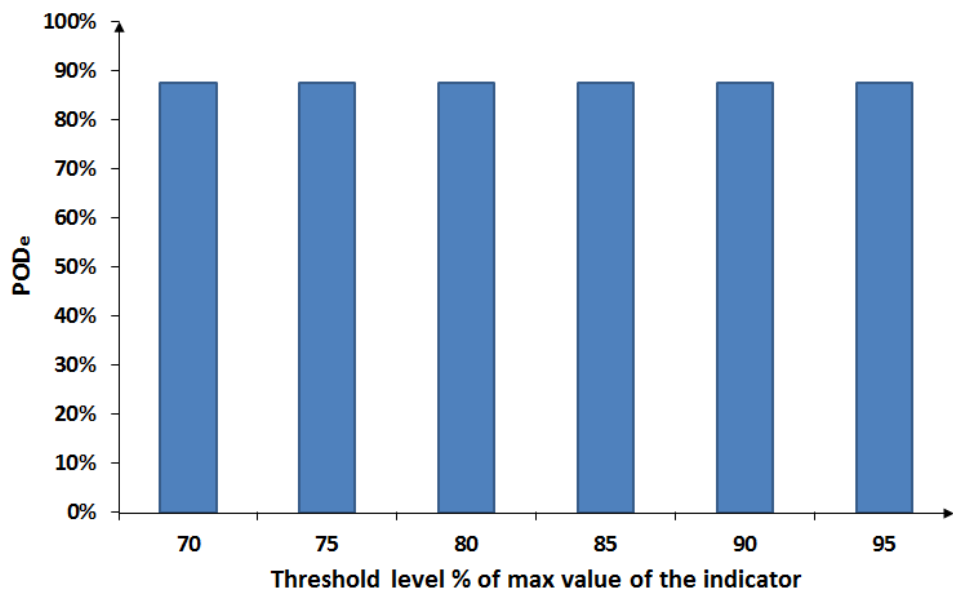


(a)  $POD_e$  in damage case 1, real experiments, strain energy indicator

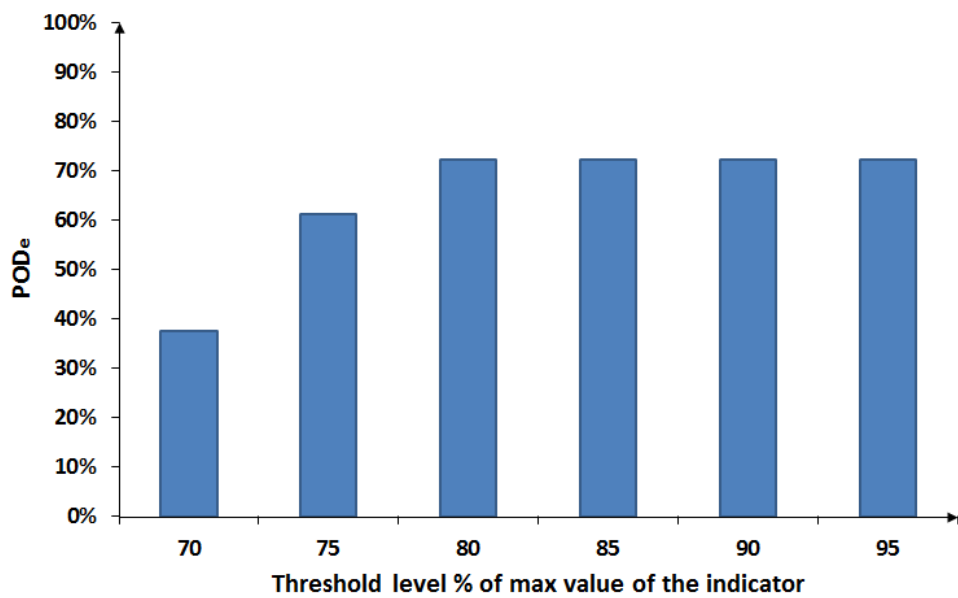


(b)  $POD_e$  in damage case 2, real experiments, strain energy indicator

Figure 8: Checking data quality



(a)  $POD_e$ , damage case 1, strain energy indicator



(b)  $POD_e$ , damage case 2, strain energy indicator

Figure 9:  $POD_e$  value, damage case 1



## 7 OUTLOOK

For future work, more partial models should be included, especially excitation, damping and noise models. More caution should be taken during the coupling of partial models in order to reduce coupling problems and their effect on the final results e.g. coupling the impulse which has a high amplitude with a very short duration with the beam could cause singularity problems. In addition, further development of the methodology for model selection and assessment based on the principles presented above is needed in order to deal with larger civil engineering structures. In addition, coupling numerical and real data should be investigated in order to solve the inverse problem.

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