The analysis of dynamic behaviour of pre-stressed systems under polyharmonic excitations

Dr. V.E. Volkova, PhD., Associate Professor, Dniepropetrovsk National University of Railway Transport, Dniepropetrovsk, Ukraine (drvev@pisem.net))

Summary

Pre-stressed structural elements are widely used in large-span structures. As a rule, they have higher stiffness characteristics. Pre-stressed rods can be applied as girders of different purpose, and as their separate parts, e.g. rods of trusses and frames. Among numerous ways of pre-stressing the compression of girders, trusses, and frames by tightenings from high-strength materials is under common application.

1 Peculiarity of dynamic behaviour of pre-stressed systems

The non-linearity of pre-stressed systems dynamic models, as a rule, is a consequent with non-linearity of their stiffness characteristics. In contrary to simple rod, the dynamic behaviour of rod pre-stressed by tightening, is connected to operating of tightenings reaction forces. The influence of a tightening on oscillating rod is reflected, at first, in operating longitudinal forces compressing each span of a rod; at second, to concentrated shear forces put in tightening attachment points; at third, to bending moments applied in cross sections of a rod. During oscillations these forces not only change their value, but also they are turn at the same segment of a tightening, by reactions which one they are [3].

Peculiarity of studied systems dynamic behaviour is:

- dependence of an oscillation frequency to amplitude;
- capability of stalling sub- and superharmonic oscillations;
- existence of great number periodic regimes at fixed frequency of excitement.

These phenomena can lead to destruction or installation of emergency regimes in dynamic systems. Strength definition and the fatigue life of pre-stressed elements correctly is impossible neglecting of their non-linearity.

2 Free oscillations of pre-stressed rods

Let's consider oscillations of a hinged rod of length l. It is suspossed, that the rod has uniform cross section. The tightening is attached to centre of gravity of end sections. The geometrical and structural schemes are given in Fig. 1.

The differential equation of free transversal oscillations of this rod has a view [1]

$$B\frac{\partial^4 y}{\partial x^4} + N\frac{\partial^2 y}{\partial x^2} + m\frac{\partial^2 y}{\partial t^2} = 0, \qquad (1)$$

where $B = EI$ is flexural stiffness; y is co-ordinate transversal displacement of a rod; x is co-ordinate along a centreline of a rod; N is reaction force of a tightening, which one is determined under the formula

$$N = N^* - \frac{EF}{2l}\int_0^l \left(\frac{\partial y}{\partial x}\right)^2 dx\,, \tag{2}$$

where N^* is controlled tension of a tightening; EF - compression stiffness of a rod; m is per unit length weight of a rod; t is time, in seconds.

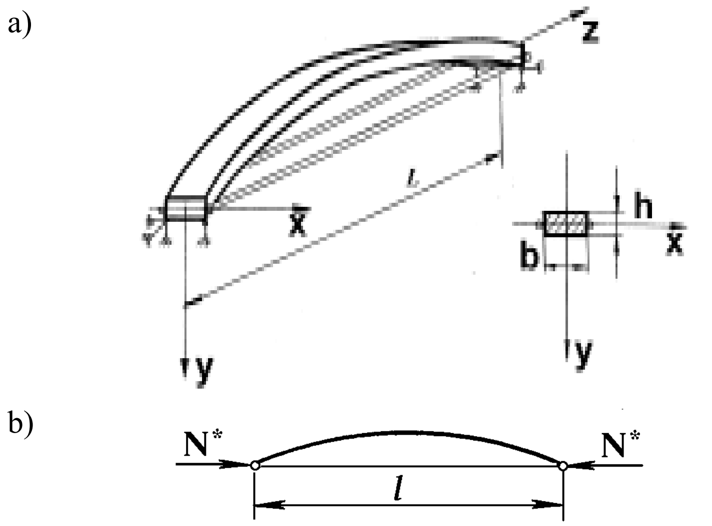

Figure 1

The geometrical and structural scheme is central pre-stressed rod

The solutions of Equation (1) can be given in form

$$y(x,t) = \phi(t)\sin\left(\frac{\pi x}{l}\right).$$

If we substitute value of tightening reaction in equation (1) and section variables, we can receive an equation of free oscillations is central of pre-stressed rood[2]

$$m\ddot{\phi} + \left(\frac{n\pi}{l}\right)^2 \left(N_n^E - N^*\right)\phi + \frac{EF}{4}\left(\frac{n\pi}{l}\right)^4 \phi^3 = 0\,. \tag{3}$$

The given equation is similar to an Duffing equation

$$\ddot{\phi} + \alpha\phi + \beta\phi^3 = 0\,. \tag{4}$$

The coefficient α can obtain both positive and negative values depending on a level of pre-stressing. In most cases, controlled tension in a tightening does not exceed Euler force ($N^* < N_n^E$) that is why coefficient α has positive value. So, investigated dynamic system is rigid. It has one steady equilibrium state. Otherwise, at $N^* > N_n^E, \alpha < 0$ - the system has three non-adjacent equilibrium state (system with double-well potential). The smoothness of deforming for such elements is upset and discontinuous transition from one steady equilibrium state to other, non-adjacent [1] at definite level of outer excitement.

3 Methods of the analysis of dynamic behaviour pre-stressed systems

Traditionally at the analysis of systems dynamic behaviour the experimental records of time processes $\phi(t)$, phase trajectories $(\dot{\phi},\phi)$ are used. But not always enough precise and full description of real objects can be constructed on their basis. As demonstrate results, these time processes and the spectral characteristics in some cases don't give the answer to question: "whether system linear or not? "[2]. It is connected with the fact that at nonresonant regimes of oscillations are close to monoharmonic. The known part of error can also be brought by an error in selection of a discretization step at processing of results.

The structure of the phase diagrams $(\dot{\phi},\phi)$ gives more complete information. It allows to make considerations about periodicity of dynamic processes and existence of singular points conforming to steady or unstable equilibrium state, but does not give a possibility to determine their position.

Other selection of phase planes parameters is also possible. The phase plane $(\ddot{\phi},\phi)$ is of great interest. It is connected that the power dependencies on it are interpreted most visually. In particular, the area, restricted by curve $\ddot{\phi}(\phi)$ equals to activity, and the circumvention of its counter-clockwise corresponds to energy loiter system. Besides the relation $\ddot{\phi}(\phi)$ is mirror symmetrical relative an axis ϕ to the graphs of restoring characteristic change [2]. The graphs $\ddot{\phi}(\phi)$ allows to establish a kind and level of non-linearity of a system.

Except for the suggested phase diagram $(\ddot{\phi},\phi)$ for the analysis of system dissipative properties the phase diagram $(\ddot{\phi},\dot{\phi})$ can be used. The main difficulty on phase diagrams $(\ddot{\phi},\phi)$ and $(\ddot{\phi},\dot{\phi})$ construction consists in necessity to exclude parameter of time t. Analytically to execute this operation it is not always possible. The majority of measuring devices is registered the changes of displacement, velocity and accelerations of investigated systems points in time. Taking sequentially the pairs of values of parameters $\ddot{\phi}(t)$ both $\phi(t)$ or $\ddot{\phi}(t)$ and $\dot{\phi}(t)$, it is possible to form this phase trajectories.

4 Technique of computing modelling

Let's investigate forced oscillations of systems drawing by non-linear differential equation of a view

$$\ddot{\phi} + \varepsilon\,\dot{\phi} + \alpha\,\phi + \beta\,\phi^3 = P\cos\omega t, \qquad (5)$$

where ε is damping coefficient; α,β - are coefficients determining nature of non-linear restoring force, P,ω - characteristic of an outer excitement. In order to obtain relations $\ddot{\phi}(t)$, $\dot{\phi}(t)$, $\phi(t)$ the software was developed. The fourth order Runge-Kutta method had been used in its. The integration step was adopted from a stability condition of a numerical integration procedure at different parameters of an equation (4). It has $\Delta t = \frac{T}{600}$, where T - a period of free oscillation, it provided stability of procedure.

It is known, that the free oscillations of non-linear systems are not monoharmonic. For an estimation of influencing of separate harmonics in the software the unit of spectral analysis had been formed. It was realised on the basis of algorithm of a fast Fourier transform.

5 Analysis of a obtaining results

The investigation of free oscillations selected for dynamic systems depicted by an equation (7), was conducted at following values of parameters: $\varepsilon = 0.5\, s^{-1}$; $\beta = 7660000\, m^{-2} s^{-2}$; the coefficient α was received for rigid systems equals $\alpha = 40.8\, s^{-2}$, amplitude of an outer excitement is $P = 1.5\, ms^{-1}$

In a fig. 2 and 3 the results of investigation for resonant and nonresonant oscillatory regimes of rigid systems are shown.

Analysing phase trajectories on planes $(\phi, \ddot{\phi})$ shown in a fig. 2, it is possible to make the conclusion about a symmetry of the elastic characteristic.

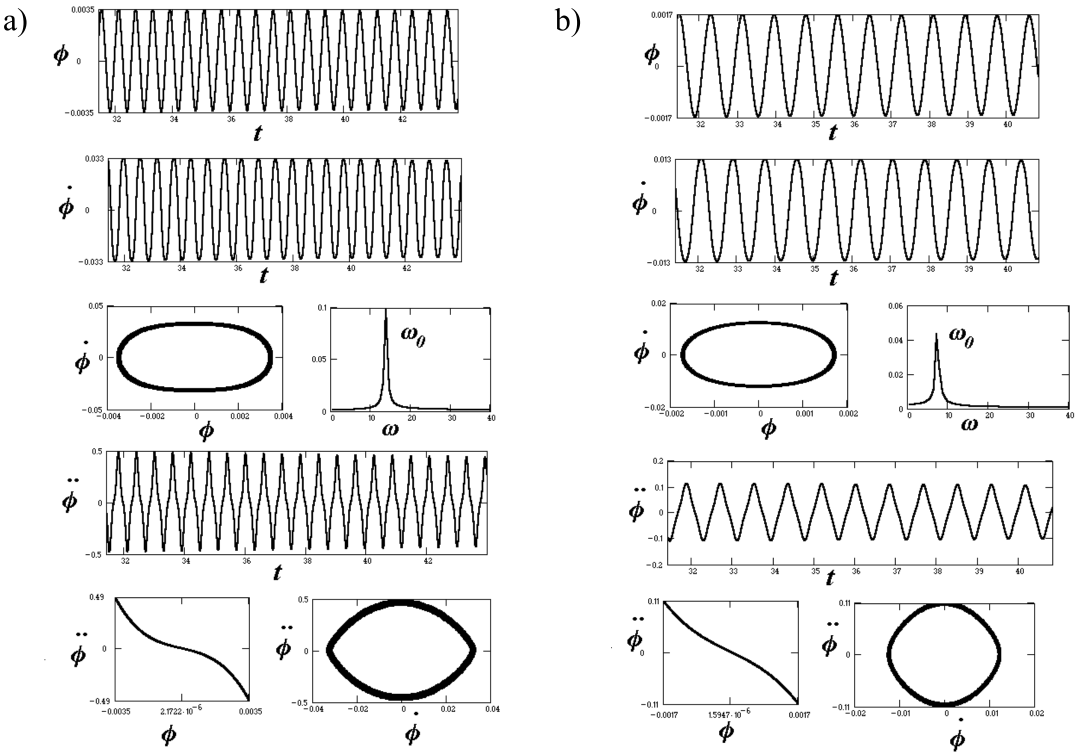

Figure 2

The time processes, spectral characteristics and phase trajectories of a "rigid" system: a) of resonance oscillations on frequency of a excitement; b) nonresonant oscillations on frequency of a excitement.

The kind of relation drawing system elastic characteristic is possible to establish by using methods of an analytical geometry. So, the phase trajectories on planes $(\phi, \ddot{\phi})$ for oscillations on frequency of a fundamental component look like a cubic parabola. The phase trajectories of resonance oscillations have the greatest number of points. That is why, they are of great interest. The method of least squares is possible to use for definition of values of parameters α and β, in the supposition of a normal distribution of measurements errors.

a) b)

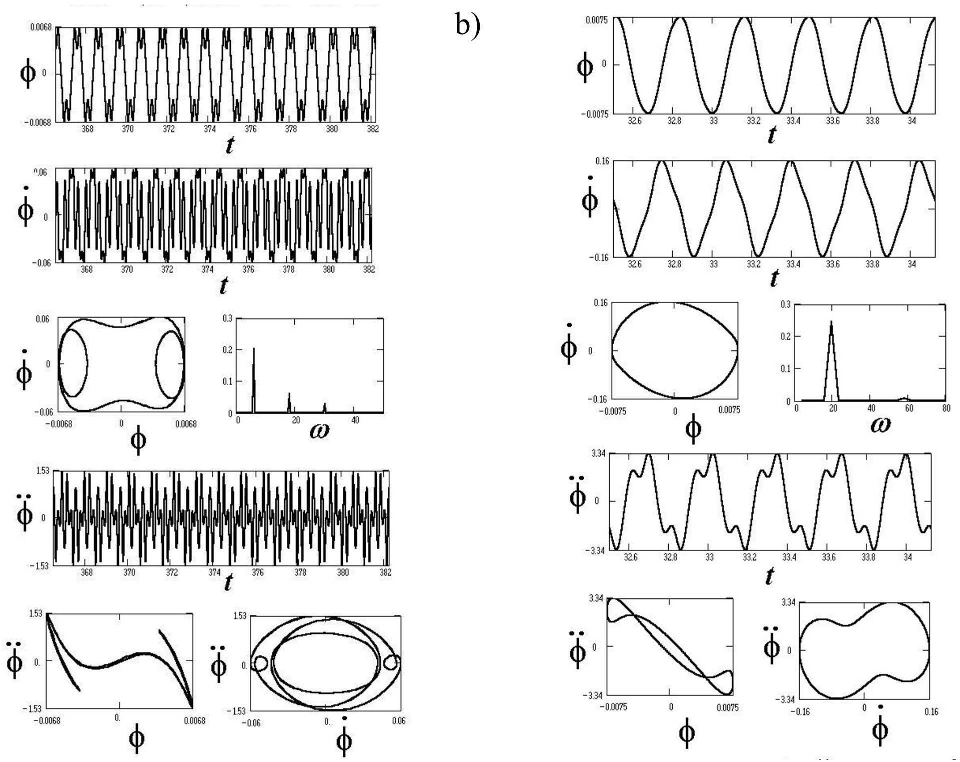

Figure 3

The time processes, spectral characteristics and phase trajectories of a “rigid” system: a) symmetrical combinative oscillations; б) of a subharmonic oscillation

a) b)

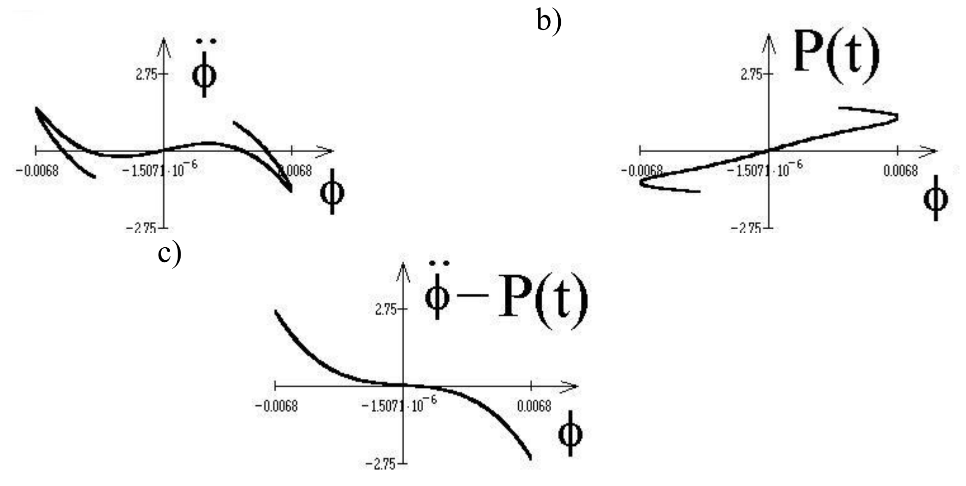

Figure 4

Graphs of response curves of symmetrical combinative oscillations

Influence of ultraharmonic oscillations and subharmonic oscillation results in change of time processes $(t, \ddot{\phi})$, they have polyharmonic nature, and to appearance of adding closed loops of

phase trajectories $(\phi, \ddot{\phi})$ and $(\dot{\phi}, \ddot{\phi})$, position and orientation which one depend on the order of harmonicses.
The analysis of system dynamic properties on the basis of time processes having polyharmonic nature is difficult. In most cases both response of a system and outer excitement measured. It is possible to simplify a procedure of investigation excluding an outer excitement from parametric relations $\ddot{\phi}(\phi)$ (see a fig. 4).

The development of qualitative methods of dynamic systems investigation suggested by the author is an effective means of the analysis and identification of dynamic systems. Simultaneous usage of all three kinds of signals, registered in time, namely displacement, velocity and acceleration allows considerably to expand possibilities of existed methods of investigation.

6 References

1. . Kazakevich M.I. & Volkova V.E. 2000. Dynamics of systems with double well potential . Dnepropetrovsk: Art-Press. (in Russian)

2. Kazakevitch M.I. & Volkova V.E. 2001 Identification of non-linear dynamic systems. Proc. 7-th International Conference Modern Building Materials, Structures and Techniques , vol.3. Vilnius: Technika :p. 145-150.

3. Oscillations in engineering: Reference book in 6 volumes, Vol.2. Moscow: Engineering