DETERMINATION OF THE TEMPERATURE OF DISSIPATIVE WARMING AND PARAMETERS OF FRACTURE IN ELASTOMERS WITH USING OF SINGULAR FINITE ELEMENTS.

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Calculation of three-dimensional bodies with cracks presents certain mathematical and computing difficulties, connected with modeling of a stress-deformed state in the vicinity of a crack. This problem can be partially solved by concentration of a grid of finite elements at the crack top, that results in the increase of dimension of a problem and time of count, especially in three-dimensional case.

Deformations and stresses near the crack front are known to have singularity of type $1/\sqrt{r}$, which can be received by displacement of intermediate knots of a square-law element on 1 / 4 length of the side in the direction to crack top. In this case jacobian of transformation of global coordinates x,y,z, to local p, q, s becomes indefinite in the crack top . Considiring that in FEM numerical integration is used and all parameters of the matrix of rigidity are determined inside the element (in points of integration), there appear no computing difficulties and usual algorithm is used.

In the given work it is offered to use the three-dimensional square-law elements , in which coordinates of knots on ribs are calculated under the following formulas [1]:

$$t_4 = \tfrac{1}{4}(3t_1 + t_6); \qquad t_5 = \tfrac{1}{4}(3t_3 + t_8) \qquad (1)$$

$$t_9 = \tfrac{1}{4}(3t_1 + t_{13}); \qquad t_{10} = \tfrac{1}{4}(3t_3 + t_{15})\ ;\ (t = x,y,z)$$

Units 1,2,3 are supposed tobe in the crack top. Isoparametric display FE, i.e. the dependence between local coordinates (p, q, s) and global coordinates (x, y, z)is recorded with use of function of the form of the same kind, as for displacement [2]. The account of displacement of the element as the rigid whole is executed on the basis of the moment scheme of finite element method [2].

For the use of such elements at the research of crackformation in constructions from nearly incompessible elastomers the threefold approximation of fields of displacements, deformations and function of volume charge [3] is supposed.

We shall consider behaviour of linear function q along the rib 1-6, (fig.1) where p=-1, s=-1. On the rib 1-6 all functions of the form are equal a zero except N1, N4, N6. Then with the account (1) the transformation of coordinates will be the following:

$$t = N_1 t_1 + N_4 \frac{1}{4}(3t_1 + t_6) + N_6 t_6 = \frac{1}{8}(1-p)(1-q)(1-s)(-p-q-s-2)t_1 +$$
$$+ \frac{1}{16}(1-p)(1-s)(1-q^2)(3t_1 + t_6) + \frac{1}{8}(1-p)(1-q)(1-s)(-p+q-s-2)t_6 . \quad (2)$$

When p= -1 and s= -1 the isoparametrical transformation will be as:

$$t - t_1 = \frac{1}{4}(1+q)^2 (t_6 - t_1), \ (t = x, y, z) \qquad (3)$$

Distance r from the top of the crack along the rib 1-6 will be determined by the ratio:

$$r^2 = (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 =$$
$$= \frac{1}{16}((x_6 - x_1)^2 + (y_6 - y_1)^2 + (z_6 - z_1)^2)(1+q)^4 \qquad (4)$$

And consequently $q \sim \sqrt{r}$. Jacobian of this transformation in the point 1 (the crack top) turns into zero, as here q= - 1.

It is similarly possible to show, that along the rib 1 - 13 change $s \sim \sqrt{r}$,

as well as change q and s along ribs 3-8 and 3-15, accordingly and in

point 3 jacobian of transformation is equal to zero.

In operating conditions the elastomer elements with surface fatigue cracks during some time preserve their technological properties. The time of growth of fatigue cracks is comparable to durability of such elements. In case of intensive mode of loading, at short-term work or at the effect of external environment the temperature of elastomers is essentially increased owing to dissipation of energy. Thermal loads exert influence on growth of cracks.

In this connection the study of processes of destruction with the account of thermopower loading is urgent.

In such cases with the increase of temperature of dissipative warming it is necessary to take into account the dependence of the physical,mechanical and reological characteristics of material on temperature.

The research of process of heat allocation at cyclic deformation of costructions from elastomers comes, as a rule, to the decision of a connected problem of thermoviscoelasticity. The algorithm of determination of temperature fields of dissipative warming in elastomers with cracks provides for realization of several stages: determination of stressed-deformed condition in nonlinear statement, calculation of parameters of the fracture mechanics, determination of temperatures of dissipative warming, calculation of the sizes of a developed crack.

In the field of crack top the elastomer undergoes large deformations. In this case the elastomers behave as a geometrically nonlinear elastic material is. The account of stress-deformed condition of the nonlinear material is done on the basis of the modified method by Newton-Êàntorovich in combination with the method of successive loading.

The size of the area, on which the average of stress intensity factor (S I F) is made, is equal to 3 / 10 of the length of the crack. In elements, lyng on the edges of crack, S I F are determined on unit displacements, and in elements, lying in front of the crack top, S I F are determined by stresses.

The speed of developement of a crack is determined with the account of the obtained temperature of dissipative warming with the use of Cherepanov 's equation [4]. On the basis of distribution S I F speeds of progress of sites of a crack front are determined. Recalculation of displacements of front sites during a step on time (or for certain quantity of loading cycles) is further made. Then a new file of coordinates is formed, taking into account geometry of a developed crack and the process of calculation is repeated.

The iterative process proceeds until the sizes of the crack achieve critical significances for the cinstruction in view or the deformations of the construction exceed the limits allowable on technological standarts.

The above stated technique of determination of parameters of crack formation and warming is realized within the framework of the computer complex of the programs KODETOM for ÐÑ and a number of problems is solved.

Problem 1. A plate with symmetric border cracks.

The sizes of the plate: a=0.5 m., b=0.2 ì., h=0.02 m. A module of shift is G= 0.07 ÌPà, Poisson 's ratio ν=0.49, stretchenal stress s = 0.01 MPa. The length of crack l=0.2 b. Opening of crack and S I F obtained in [5], accordingly are equal: $\Delta / 2 \cdot 10^3 = 4.98$ m, $K_I \cdot 10^3 = 4.37$ MN / m $^{3/2}$.

In table 1 the results of numerical accounts on determination of opening of a crack and stress intensity factor K_I for linear, Lagrauge's square-law , square-law singular, Serendip's square-law and Serendip's singular of elements are indicated at various grids of finite elements. The analysis of results shows, that the use of singular elements improves the results in comparison with usual square-law or linear. Besides the application of Serendip's family element reduces dimension of the problem and time of the count.

Problem 2. A thermostrained condition of a rubber-metall element of BRM 101 type at cyclic deformation of shift.

The size of rubber element: length of 0.1 m., width of 0.06 m., height of 0.035 m., amplitude of loading Δ = 0.015, frequency of loading 8.7Hz ., rubber model 51-1562, shift module G = 0.51 MPa, Poisson's ratio ν = =0.499, viscosity of fracture $\hat{E}_{Ic} = 0.63043$ MN/·m$^{3/2}$.

The count of the construction made with a put surface crack in the form of a rectangular incision with the size 10 x10 mm. In such an intensive mode of loading the crack grew up to sizes, comparable to the sizes of construction at 32 hours of loading. It is characteristic that the speed of spreading of the crack deep inside the construction is less than that on the surface. It is explained by the distribution of S I F (fig. 1).

At the same time the intensity of internal sources of heatproducing and temperature of dissipative warning of a prismatic element of the shift with various corners of inclination of a surface crack were determined (fig. 2,3). The essential burst of significances of heat-producing sources near to a crack speaks that in the given area there are large gradients of deformations. However, the maximum value of temperature is neverth less observed in the centre of the body, which is connected with thermomechanic effects.

With the change of the corner of the surface crack from the vertical axis the character of distribution and significance, of sources, as well as the of temperature of dissipative warming changes.

Plate with symmetric border cracks Table 1.

Parameter	Grid	Element				
		linear	quadratic		serendip	
			usual	singular	usual	singular
crack openig $\Delta/2 \cdot 10^3$m	3×9×7	1.99	2.77	2.95	2.75	2.88
	3×13×9	2.27	3.39	3.52	3.39	3.44
$K_I \cdot 10^3$ $MN/m^{3/2}$	3×9×7	7.72	7.27	7.56	7.27	7.8
	3×13×9	6.13	5.87	6.32	6.00	6.92

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Óêðàèíà, 348008, Ëóãàíñê - 8, Ëóãàíñêèé ñåëüñêîõîçÿéñòâåííûé èíñòèòóò, êàôåäðà âûñøåé è ïðèêëàäíîé ìàòåìàòèêè.

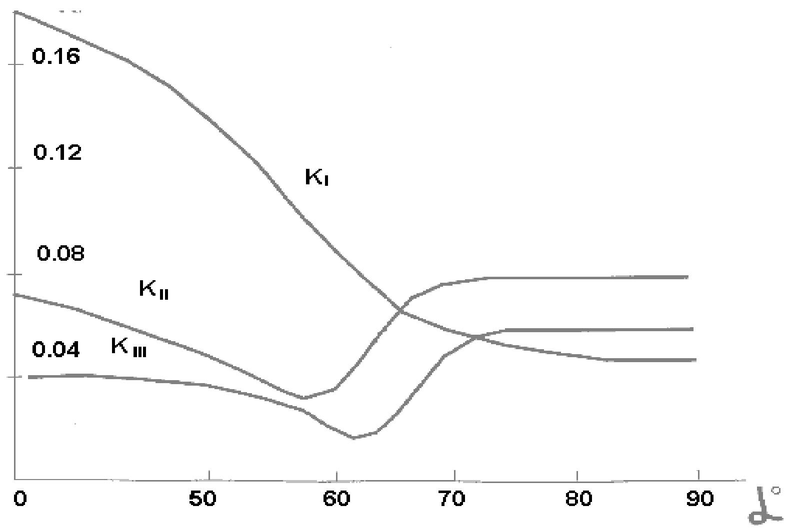

Fig. 1

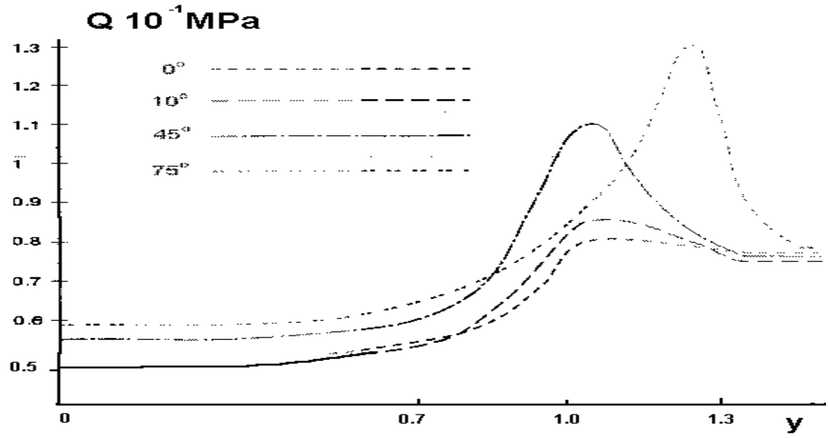

Fig. 2

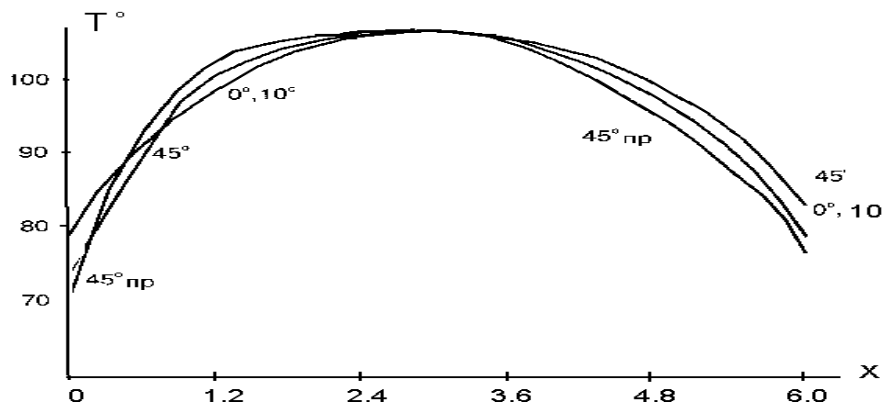

Fig. 3