Discussion of "Estimation of one-dimensional velocity distribution by measuring velocity at two points" by Yeganeh and Heidari (2020)

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Abstract The concept of information entropy together with the principle of maximum entropy to open channel flow is essentially based on some physical consideration of the problem under consideration. This paper is a discussion on Yeganeh and Heidari (2020)'s paper, who proposed a new approach for measuring vertical distribution of streamwise velocity in open channels. The discussers argue that their approach is conceptually incorrect and thus leads to a physically unrealistic situation. In addition, the discussers found some wrong mathematical expressions (which are assumed to be typos) written in the paper, and also point out that the authors did not cite some of the original papers on the topic.

1. **Discussion:** The authors claim to have proposed a new approach for measuring vertical distribution of velocity in open channels using entropy concept. They have considered three different measures of entropy, which already exist in literature, to carry out the analysis presented in their paper. The main objective of this discussion is to show that the new approach proposed by the authors is not correct conceptually. Further, there are plenty of wrong mathematical expressions (which the discussers believe to be typos or written without any careful consideration) written in the paper, and surprisingly the authors did not cite some of the earlier papers on the same concepts. Each of these issues is discussed in what follows.

   First off, the authors use the concept of Renyi entropy for deriving the one-dimensional velocity distribution in open channels. Unfortunately, they do not cite any of the original papers [1,2] in the entire texts of the paper. Also, the original works on Tsallis entropy-based velocity distribution [3,4] have not been referred to at the proper places in the text of the paper. The authors cited the work Luo and Singh [3] once when they discussed the feasible range of the Tsallis entropy index. Importantly, except for the new approach (equations) proposed by the authors, all the equations derived in their paper are similar to those derived in the earlier papers; even the discussion on the Renyi and Tsallis entropy indexes are same. However, the discussers' objective is not to question this particular issue but to comment on the new approach proposed by the authors, which is discussed in the next paragraph.

The concept of information entropy together with the principle of maximum entropy was introduced to the area of fluvial hydraulics by Chiu [5]. Here, we will discuss the concept based only on the Shannon entropy in relation to the authors' paper. Similar conclusions can be made for both the Tsallis and Renyi entropies. The Shannon entropy for the streamwise velocity component was given by Eq. (6), and the constraints chosen were written in the form of Eqs. (4) and (5). Chiu [6] showed that the first three raw moments (in terms of constraints) physically represent the hydrodynamic transport of mass, momentum, and energy through a cross-section of open-channel flow. For simplicity, the constraints based on the total probability and the first-order moment can be taken into account for the derivation of the velocity profile. Considering these two constraints and applying the maximum entropy principle, one can derive the most probable probability density function (PDF) in the form of Eq. (8). Now, based on the flow characteristics and channel geometry, the cumulative distribution function (CDF) can be proposed as given by Eqs. (1) and (2). Finally, connecting this hypothesized CDF and the CDF obtained by the maximum entropy principle, one can obtain the velocity equation (11). The analysis can be further simplified by defining an entropy parameter [5] M, as followed in the paper. The final velocity equation depends on the single parameter M as the maximum velocity u_{max} is available for a particular vertical or a cross-section of an open channel. The parameter M can be determined from an implicit relation Eq. (11) with the given values of $\bar{u}/u_{max}$ where $\bar{u}$ is the mean velocity. In accordance with the theoretical development of the entropy-based approaches, it is clear that the parameter M is fixed along a particular vertical (1D) or a cross-section (2D) based on the consideration. Indeed, the authors state these things in their paper, such as '…u_{max} is the maximum velocity in a river cross-section', '…Φ is the constant ratio… for a cross-section', etc. But then, they propose a new approach for determining the parameter M and the maximum velocity u_{max} given by Eqs. (15) and (16) considering velocity in two different depths in Eq. (14) provided by Eqs. (17) and (18). Hence, without any surprise, this consideration leads to a physically unrealistic situation where the maximum velocity and the value of the parameter vary for a particular vertical considering some pairs of two different depths and the corresponding velocities (refer to Fig. 1 in their paper). The same mistake can be found in Figs. 3 and 5, which provided in their paper, for Renyi and Tsallis entropy-based approaches, respectively. Henceforth, the discussers believe that there is no point in discussing the paper as the remaining analysis is based on a conceptually incorrect idea.

The M parameter has been proved to be a characteristic of river site and doesn't change with the flow [7]. This aspect is of paramount importance for monitoring discharge during high flow and there is no need to estimate M for each velocity profile across the river site. The worth of M consists right in the fact that it can be estimated by leveraging the pairs $(\bar{u}, u_{max})$ of velocity dataset, even if the dataset is bounded to low flow. In the case of ungauged site the approach proposed by [7] and better detailed by [8] can be applied.

Also, the discussers want to point out some wrong mathematical expressions written in the paper, which are assumed to be typos but can be misleading for readers. Eq. (23): the term $(-\lambda_1 - \lambda_2)^{\frac{\alpha}{\alpha-1}}$ should be replaced by $(-\lambda_1 - \lambda_2 u_{max})^{\frac{\alpha}{\alpha-1}}$; Eq. (35): A, B, and C should be replaced by A_r, B_r, and C_r, respectively; Eq. (40): the term $(\lambda_* + \lambda_2)^{\frac{m}{m-1}}$ should be replaced by $(\lambda_* + \lambda_2 u_{max})^{\frac{m}{m-1}}$; Eq. (45): ξ_1 in the denominator of the first term in the right-hand side should be ξ_2. Apart from these, there are some statements in the paper which might be conceptually incorrect. For example, the authors write 'The sum of the probabilities of all possible states of an event is always equal to one…' which is true for *discrete* case only, not *continuous*; 'Tsallis [23] indicated that, for $m = 1$,…' which is not true as the Tsallis entropy is undefined for $m = 1$, it should be $m \to 1$. Finally, in the conclusions section, the authors use the term 'reinvestigated' but surprisingly, throughout the paper, they have not cited most of the original works.

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