A New Efficient Concept for Elasto-plastic Simulations of Shell Responses

U. Montag

Institute for Statics and Dynamics, Ruhr-University Bochum,
Universitätsstrasse 150, D-44780 Bochum, Germany

Abstract

The present paper introduces a highly efficient numerical simulation strategy for the analysis of elasto-plastic shell responses. An isoparametric Finite Element, based on a Finite Rotation Reissner-Mindlin shell theory in isoparametric formulation, is enhanced by a Layered Approach for a realistic simulation of nonlinear material behaviour. A material model including isotropic hardening effects is embedded into each material point. A new, highly accurate integration scheme is combined with consistently linearized constitutive relations in order to achieve quadratic rate of convergence. A global Riks-Wempner-Wessels iteration scheme enhanced by a linear Line-Search procedure has been used to trace arbitrary deformation paths. A numerical example shows the efficiency of the present concept.

1 Introduction

Nonlinear computer simulations of realistic systems, like the crash-worthiness analysis of a car or the snap-through response of a shell, are dominated by time-consuming computations for consideration of nonlinear material behaviour, especially when layered shell models are applied. The coincidence of all components, as the chosen Finite Element, the numerical integration scheme of the material model, the consistently linearized constitutive tensor, and the path-following strategy decide upon the possible size of load steps, the convergence behaviour and the accuracy of the computed deformation path.

The paper starts with the presentation of a Reissner-Mindlin type shell theory including arbitrary large deformations in isoparametric formulation [1] as the basis for the formulation of corresponding finite elements. To avoid shear-locking effects, the assumed-strain concept [2] has been employed to obtain the locking-free finite element T5-IAS4. For the consideration of nonlinear material laws, two major concepts can be found in the literature: the stress resultant model or the Layered Approach. The first concept, employed e.g. in [3], is advantageous because of its notational simplicity and the assurance of quadratic convergence in a neighbourhood of the exact solution. The Layered Approach, used in the present model, allows a much more accurate prediction of the stress distribution throughout the shell thickness. The insurance of quadratic convergence near to the exact solution is also guaranteed for the present model. The tracing of nonlinear deformation paths, even in a post-peak range, is performed by the Riks-Wempner-Wessels iteration scheme. To ensure convergence for very large load steps, a Line-Search procedure was implemented into the global iteration scheme.

The highly accurate solution of the initial value problem, formulated by nonlinear constitutive relations including elasto-plastic material behaviour, preserves an exact tracing of nonlinear deformation paths even if large load steps are applied. The starting point for the derived algorithm is represented by the principle of maximum plastic dissipation. The integration of the nonlinear constitutive relations is transformed into a constrained optimization problem. This problem can be solved with any prescribable accuracy either by direct nonlinear programming optimization methods [4] or, after transformation, by a Newton procedure, in which the plastic multiplier presents the only independent parameter. To ensure quadratic convergence in a close neighbourhood to the exact solution, a consistent linearization of the constitutive tensor has been performed on the one hand for the in-plane components and on the other hand for the component due to shear deformations.

Efficiency and accuracy of the presented simulation strategy is shown by a numerical example, performed within the finite element system FEMAS2000 [5] developed at the Institute for Statics and Dynamics.

2 Finite-Element Formulation and Numerical Path-Tracing

2.1 Finite-Rotation Shell Theory in Isoparametric Formulation

In the following, a Reissner-Mindlin-type shell theory applicable to the finite-rotation analysis of arbitrary structures is presented. The equations are derived in an isoparametric formulation, which enables the derivation of corresponding finite elements.

Let $\mathring{\mathbf{r}} = \mathring{\mathbf{r}}(\theta^\alpha)$ be the position vector determining the undeformed surface $\mathring{F}$ of a shell structure with respect to the unit vectors $\mathbf{i}^i = \mathbf{i}_i$ associated with an orthogonal Cartesian reference frame X^i, where θ^α denote curvilinear coordinates of the given surface $\mathring{F}$. Using isoparametric coordinates with limit values $\xi^\alpha = 0,1$, we approximate the position vector $\mathring{\mathbf{r}}$ by

$$\mathring{\mathbf{r}} = \mathring{X}^i(\xi^\alpha)\mathbf{i}_i \approx \sum_{L=1}^{N} f^L \mathring{X}^{iL} \mathbf{i}_i , \tag{1}$$

where f^L are shape functions and $\mathring{X}^{iL}$ the given values of $\mathring{X}^i$ at the points $\mathring{P}^L$ (L=1,...,N). To introduce 2-D strain variables we approximate the position vector $\mathbf{r}^*$ associated with the deformed position of an arbitrary point of the shell continuum by a linear function of the thickness coordinate ξ^3 as

$$\mathbf{r}^* = \mathbf{r} + \xi^3 \mathbf{a}_3 , \tag{2}$$

where $\mathbf{r}$ is the position vector of the deformed middle surface F and $\mathbf{a}_3$ the base vector associated with the deformed coordinate line ξ^3. Both vectors $\mathbf{r}$ and $\mathbf{a}_3$ are defined with respect to the global reference frame $\mathbf{i}^i = \mathbf{i}_i$, thus

$$\mathbf{r} = X^i \mathbf{i}_i , \quad \mathbf{a}_3 = A^i \mathbf{i}_i . \tag{3}$$

The rotational variables ψ_α are introduced as primary variables. Thus, the unit vector $\mathbf{a}_3 = A^i \cdot \mathbf{i}_i$ is determined by

$$A^1 = \sin\psi_1 \cos\psi_2 , \ A^2 = \sin\psi_1 \sin\psi_2 , \ A^3 = \cos\psi_1 . \tag{4}$$

The deformed shell continuum is described by five independent variables X^i and ψ_α. Using the standard Finite-Element procedure, the incremental equations can be derived and transformed into nonlinear element matrices.

Pure displacement-based models on the basis of shear deformation theories are sensitive to shear locking. To avoid this deficiency, the assumed-strain concept, which has been successfully employed in previous works [2], is used for the interpolation of the transverse shear strains. Further characteristics of the developed four-noded element T5-IAS4 are summarised in [1].

2.2 Layered Finite-Element Formulation

The shell theory of the previous section will be enhanced to consider nonlinear material behaviour. For the physically nonlinear simulation in a highly accurate way, the Layered Approach has been introduced into the Finite Element T5-IAS4. This simulation strategy enables - in contrast to the stress-resultant strategy - the prediction of stresses throughout the shell thickness.

In order to compute the required internal forces, the integration of the stresses will be transformed into a summation over the thickness and the incremental internal forces are evaluated from the consistently linearized constitutive tensors of the layers.

2.3 Path-following Strategies enhanced by Line-Search Procedures

For the solution of the global Finite Element equation the Standard Newton-Raphson as well as the Riks-Wempner-Wessels iteration scheme have been employed. These global iteration schemes have been enhanced by a linear Line-Search procedure to ensure convergence for greater load steps. In the present Line-Search procedure, only interpolations are selected due to possible "dangerous" extrapolations, which may destroy the convergence of the iteration scheme.

3 Numerical Formulation for Elastoplastic Analysis

3.1 Closest-Point Projection Algorithm

The integration of the elastoplastic constitutive model employs a closest point projection scheme which, for linear isotropic hardening, may be expressed in energy norm by means of the following functional presenting the complementary plastic dissipation [6]

$$\chi\left(\sigma^{ij},\overset{+}{\gamma}{}^{p}_{eqv}\right)=\frac{1}{2}\left({}^{i}\sigma^{ij}_{trial}-\sigma^{ij}\right)G_{ijkl}\left({}^{i}\sigma^{kl}_{trial}-\sigma^{kl}\right)+\frac{1}{2}K\left(\overset{+}{\gamma}{}^{p}_{eqv}\right)^{2}\ , \tag{5}$$

where ${}^{i}\sigma^{ij}_{trial}$ represents the trial elastic state at time ${}^{i}t={}^{i-1}t+\Delta t$, computed by

$${}^{i}\sigma^{ij}_{trial}={}^{i-1}\sigma^{ij}+C^{ijkl}\,{}^{i}\overset{+}{\gamma}_{kl}\,,\quad {}^{i}\overset{+}{\gamma}_{kl}={}^{i}\gamma_{kl}-{}^{i-1}\gamma_{kl}\,. \tag{6}$$

Herein, C^{ijkl} abbreviates the elastic material tensor [7], G_{ijkl} and K represent the inverse elastic material tensor and the plastic modulus, respectively. $\overset{+}{\gamma}{}^{p}_{eqv}$ is the equivalent plastic strain increment.

According to the plastic dissipation principle, the actual state of the stress σ^{ij} and internal variable $\overset{+}{\gamma}{}^{p}_{eqv}$ is that one among all possible states, satisfying the yield condition, that minimizes the functional (5). It can be interpreted geometrically as the closest point projection of the trial state onto the yield surface and solved by direct nonlinear optimization procedures [8].

By introduction of the Lagrangean functional

$$L\left(\sigma^{ij},\gamma^{p}_{eqv},\lambda\right)=\chi\left(\sigma^{ij},\gamma^{p}_{eqv}\right)+\lambda\,F\left(\sigma^{ij},\gamma^{p}_{eqv}\right)\ , \tag{7}$$

the constrained minimum principle is transformed into the unconstrained problem which is formulated in standard Kuhn-Tucker form in [8,9]. This optimality condition, employing the associativity of the flow and hardening rules and the loading / unloading conditions, yields the relations for the stress tensor and the equivalent plastic strain increment. This resulting equation may be solved for ${}^{i}\lambda$. In this paper, the iterative solution procedure employing Newton's method is applied.

After determination of the plastic multiplier, the updated values of the stresses can be computed, and the equivalent plastic strain as well as the stresses are obtained. To avoid spurious unloading during the iteration process, all state variables are updated with respect to the previous equilibrium state.

3.2 Consistent Tangent Modulus

The elastoplastic tangent modulus [10] is derived by linearization of the update algorithm presented in the previous section on the one hand for the in-plane components of the stress tensor and on the other hand for the stresses due to shear deformations. This procedure guarantees a quadratic convergence behaviour in the neighbourhood of the exact solution for the applied Reissner-Mindlin type theory [8].

The differential relationships between the internal forces and the strain components leads to the required internal forces, from which the stiffness matrices can be calculated.

4 Numerical Applications

4.1 Shallow Cylindrical Shell under Point Load

As a numerical example, the geometrically nonlinear elastoplastic response of a shallow cylindrical shell under point load will be studied. The shell is hinged at the longitudinal edges and free along the curved boundaries. Geometry, finite element mesh and material data are presented in Figure 1.

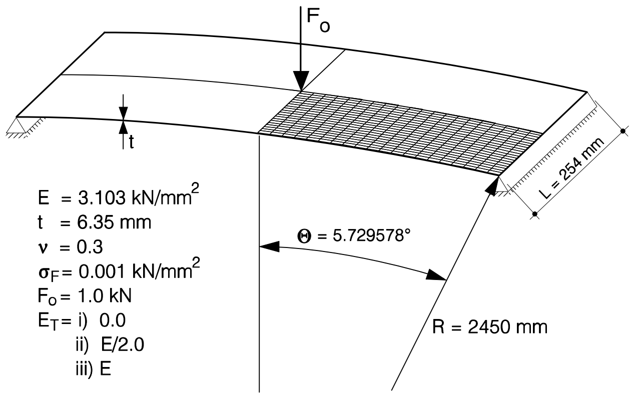

Figure 1. Geometry, Finite Element Mesh and Material Data for Shallow Cylindrical Shell

A finite element mesh of 20x20 elements is used to discretize one quarter of the structure. The computation is performed for different isotropic hardening parameters expressed by different tangent moduli E_T. The dependence of the vertical displacement under the force F_0 on the load factor is plotted in Figure 2.

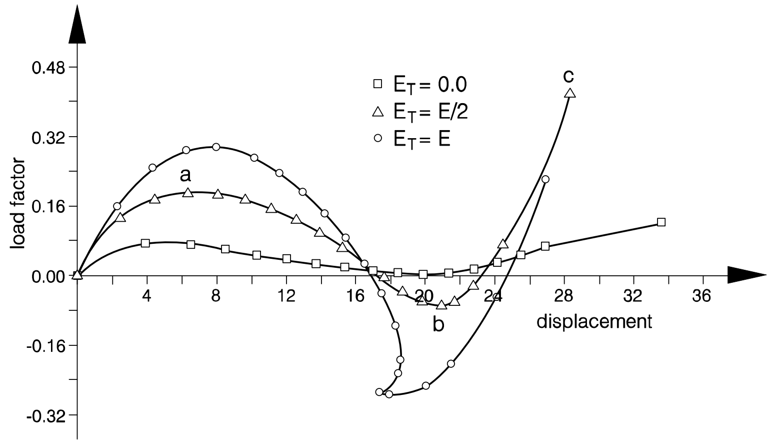

Figure 2. Load - Displacement Curves for Different Hardening Parameters

This figure compares the curves computed by the present algorithms using elastic-ideal plastic $(E_T = 0.0)$ and isotropic hardening $(E_T = E/2)$ material models with those obtained by elastic analysis $(E_T = E)$. As it may be seen, the structure exhibits elasto-plastic snap-through behaviour in contrast to the elastic response where snap-through as well as snap-back phenomena are pronounced. The propagation of plastic zones throughout the shell thickness for several sections, calculated for isotropic hardening material at various load levels noted in Figure 2, is shown in Figure 3.

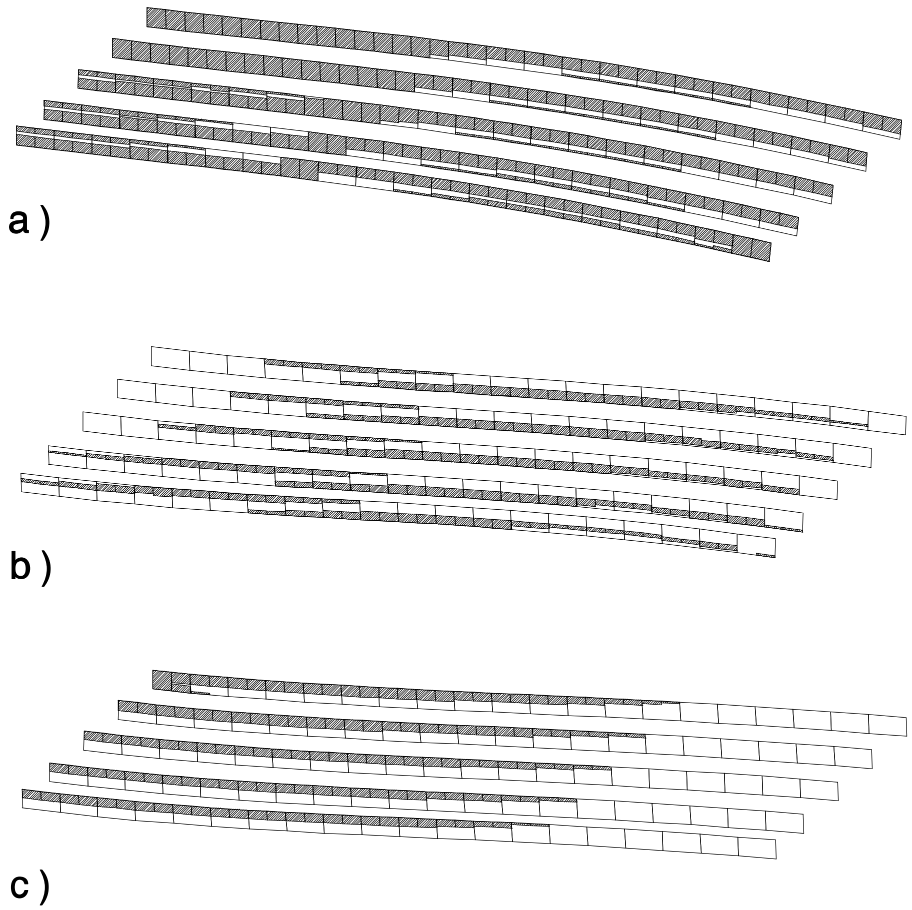

Figure 3. Spread of Plastic Zones throughout Shell Thickness for Three Load Factors noted in Figure 2

The sections are traced by planes parallel to the longitudinal edges of the shell. Evidently, the redistribution of plastic zones becomes apparent again.

5 Summary and Conclusions

For the analysis of arbitrary discretized shell structures, an efficient numerical simulation strategy including geometrically and physically nonlinear effects has been presented.

In the beginning, a Finite-Rotation shell theory allowing constant shear deformations across the shell thickness is presented in an isoparametric formulation. The assumed-strain concept enables the derivation of a locking-free finite element. The Layered Approach has been applied to ensure a sufficiently precise prediction of the spread of plastic zones throughout the shell thickness. A Riks-Wempner-Wessels global iteration scheme enhanced by a Line-Search procedure ensures the tracing of nonlinear deformation paths under rather great load steps even in the post-peak range.

A material model, including isotropic hardening, has been employed to describe elasto-plastic material behaviour. A new Operator-Split return algorithm ensures a highly exact solution of the initial-value problem even for great load steps. The combination with consistently linearized constitutive equations ensures quadratic convergence in a close neighbourhood to the exact solution.

Finally, an example demonstrates the accuracy and numerical efficiency of the presented algorithm in the pre- and post-peak range.

Acknowledgement

The author would like to express his gratitude to o. Prof. Dr.-Ing. Dr.-Ing. E.h. W. B. Krätzig for his valuable scientific assistance.

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