Identification of the thermal properties of concrete for the temperature calculation of concrete slabs and columns subjected to a standard fire— Methodology and proposal for simplified formulations

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ABSTRACT

The fire resistance of concrete members is controlled by the temperature distribution of the considered cross section. The thermal analysis can be performed with the advanced temperature dependent physical properties provided by EN 1992-1-2. But the recalculation of laboratory tests on columns from TU Braunschweig shows, that there are deviations between the calculated and measured temperatures. Therefore it can be assumed, that the mathematical formulation of these thermal properties could be improved. A sensitivity analysis is performed to identify the governing parameters of the temperature calculation and a nonlinear optimization method is used to enhance the formulation of the thermal properties. The proposed simplified properties are partly validated by the recalculation of measured temperatures of concrete columns. These first results show, that the scatter of the differences from the calculated to the measured temperatures can be reduced by the proposed simple model for the thermal analysis of concrete.

1. Introduction

The fire resistance of reinforced concrete cross sections is mainly determined by the temperature dependent material properties. Therefore the accuracy of the calculated temperatures is crucial for the mechanical analysis of the considered member. Achenbach and Morgenthal perform a global sensitivity analysis of fire exposed reinforced concrete walls and columns [1,2]. The results indicate, that the uncertainty of the thermal analysis contributes the biggest part to the scatter of the results for concrete compression members subjected to a standard fire.

The recalculation of measured temperatures of concrete columns using the material properties of EN 1992-1-2 [3] reveals, that the temperatures at the surface are overestimated by calculation, while the calculated temperatures at the center are lower compared to the measured results [1]. The mean ratio of the calculated to the measured temperatures $\eta_t = \theta_{cal}/\theta_{exp}$ [-] is $\mu = 0.9$ with a standard deviation of $\sigma = 0.3$ for the examined laboratory tests.

The results of the recalculation of measured temperatures indicate, that the parameters of the thermal analysis could be improved to increase the accuracy of the results. The optimization of the formulation of the thermal properties of concrete for concrete slabs and columns heated by a standard fire is described in this paper. In Section 2, the physical model of the thermal analysis and the involved parameters are described. The identification of the most influential parameters – which are used for optimization – is performed in Section 3. The applied methods for the nonlinear optimization and the results are discussed in Section 4. The proposed simplified thermal properties are partly validated by the recalculation of laboratory tests on columns in Section 5.

2. State of knowledge

The temperature distribution in a concrete wall, heated on both surfaces as displayed in Fig. 1, is controlled by the differential equation [4]

$$\frac{\partial T(x, t)}{\partial t} = \frac{\lambda}{\rho \cdot c_p} \cdot \frac{\partial^2 T(x, t)}{\partial x^2}, \tag{1}$$

with λ= thermal conductivity [W/m K], ρ= density [kg/m³] and c_p= specific heat [J/kg K]. The boundary conditions at the surface are described by the heat flux q [W/m²]. With the surface temperature of the wall T_w [K] and the temperature of the heated gas T_g [K] these conditions are
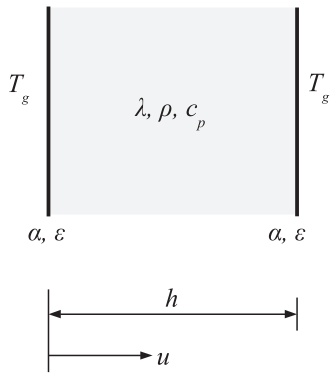
Fig. 1. Concrete wall, heated on both surfaces.

$$q = -\lambda \cdot \frac{\partial T_w}{\partial x} \qquad (2)$$

for conduction and

$$q = \alpha \cdot (T_g - T_w) + \varepsilon \cdot \sigma \cdot (T_g^4 - T_w^4) \qquad (3)$$

for convection and radiation, with α= coefficient of heat transfer [W/m^2 K], ε= emissivity [-] and σ= Stephan-Boltzmann constant [W/m^2 K^4]. The physical properties λ, ρ and c_p are temperature dependent and the above mentioned equations can only be solved in that case with numerical methods with the initial condition $T_g = T_w = T(x, t = 0) = (20 + 273.15)$ K.

The heat transfer from the heated gas to the concrete surface is mainly determined by radiation [5]. The corresponding parameter ε contains the emissivity of the flame and the surface [5,6] and is temperature dependent. The proposed constant value $\varepsilon = 0.7$ [-] for concrete members according to EN 1992-1-2 [3] is a simplification and derived from the recalculation of laboratory tests [6]. The coefficient of heat transfer α is dependent from the velocity of the heated gas at the surface [5] and describes the heat flux due to convection. The recommended constant values [7] – $\alpha = 25$ [W/m^2 K] for fire exposed and 4 [W/m^2 K] for unexposed surfaces – are also a simple approach.

The physical properties λ, ρ and c_p – determined by different scientists – show a remarkable scatter, which is caused by different experimental methods [8,9]. The heat transfer in the concrete wall is described by the thermal diffusivity $a = \lambda/(\rho \cdot c_p)$, which means that all variables are put together and the scatter of the different physical properties can be "equalized". This can also be seen in the published values for a [8,9].

The temperature dependent functions for the physical properties λ, ρ and c_p given in EN 1992-1-2 [3], must be understood as a compromise among the involved specialists [10]. The lower limit for λ has been derived from the recalculation of concrete members [10], while composite members have been used for fitting the upper limit [11].

3. Sensitivity analysis

3.1. Applied methods

A Monte Carlo simulation [12] of a concrete wall, heated on both sides by a standard fire according to EN 1991-1-2 [7], is set up. The wall is displayed in Fig. 1 and the parameters are given in Table 1. In lack of more detailed statistic key data for the thermal properties, each stochastic variable is assumed to be normally distributed with the nominal value as mean value μ and a coefficient of variation $v = \sigma/\mu = 0.1$ (Table 2). The symmetric normal distribution has been chosen to avoid any preferences, which may be caused by the choice of an asymmetric distribution, e. g. a log-normal distribution. It is also assumed, that there is no correlation between the variables. The temperature dependent physical properties and the gas temperature

Table 1
Parameters of the sensitivity analysis.

parameter	unit	value
height: h	[cm]	10
fire duration: t_f	[min]	30
heat transfer coefficient: α	[W/m^2 K]	25
emissivity: ε	[-]	0.7
density: ρ(20 °C)	[kg/m^3]	2400
moisture content: u	[%]	1.5
thermal conductivity: λ	[W/m K]	lower limit

Table 2
Basic variables of simulated walls (DET=deterministic, N=normal distribution).

variable	distribution	description
h	DET	Table 1
θ_g	N	EN 1991–1–2, standard fire
ε	N	Table 1
α	N	Table 1
λ	N	Table 1
ρ	N	Table 1
c_p	N	EN 1992–1–2, u acc.Table 1
X_t	N	model uncertainty

θ_g are multiplied by X_i, which is also normally distributed with $\mu = 1.0$ and v=0.1. One X_i is generated for each variable. The uncertainty of the physical model, described in Section 2, is modeled by the variable X_t and contains all uncertainties, which are not covered by the scatter of the other variables. These uncertainties are for instance: the radiation conditions in the testing furnace, the error in temperature measurement of the thermocouples and the possible incompleteness of the mathematical model. It is assumed, that these uncertainties can be described by normally distributed ($\mu = 1.0$ and v=0.1), multiplicative variable X_t. The calculated temperatures are multiplied by X_t to consider these model uncertainties.

A number of 5000 samples is generated and the temperature distributions for $t_f = 30$ min are calculated. The results for a distance to the surface u=0, 2.5 and 5.0 cm are evaluated in the sensitivity analysis. Spearman's rank correlation coefficients [13] and first order Sobol [14] indices are used for the assessment of the sensitivities of the results against the stochastic variables.

The Spearman rank correlation coefficient r_{Si} is a measure for the correlation between the values oft the considered variable X_i and the results Y [13]. Values $|r_{Si}| \approx 0$ indicate, that there is no correlation between the values of the considered variables. Results $|r_{Si}| = 1$ show, that there is a full linear or nonlinear monotonic correlation – all other values need interpretation. Values of $r_{Si} \geq 0$ show, that increasing values for X_i lead to increasing values for Y (positive correlation).

The first order Sobol indices are a variance based measure for the sensitivity. They are based on the assumption, that a completely unknown function y can be described by a function $f(x)$ with terms of increasing dimensionality [15]:

$$f(x) = f_0 + \sum_i f_i(x_i) + \sum_{i<j} f_{ij}(x_i, x_j) + \cdots + f_{12\ldots n}. \qquad (4)$$

The variance of each term is $V_i = V(f_i(x_i))$, $V_{ij} = V\left(f_{ij}(x_i, x_j)\right)$, ... and it can be concluded that the total variance is described by

$$V = \sum_i V_i + \sum_{i<j} V_{ij} + \cdots + V_{12\ldots n}. \qquad (5)$$

The ratio

$$S_i = \frac{V_i}{V} \qquad (6)$$

is the so called first order Sobol index and is a measure for the contribution of the variance of one single variable to the total variance
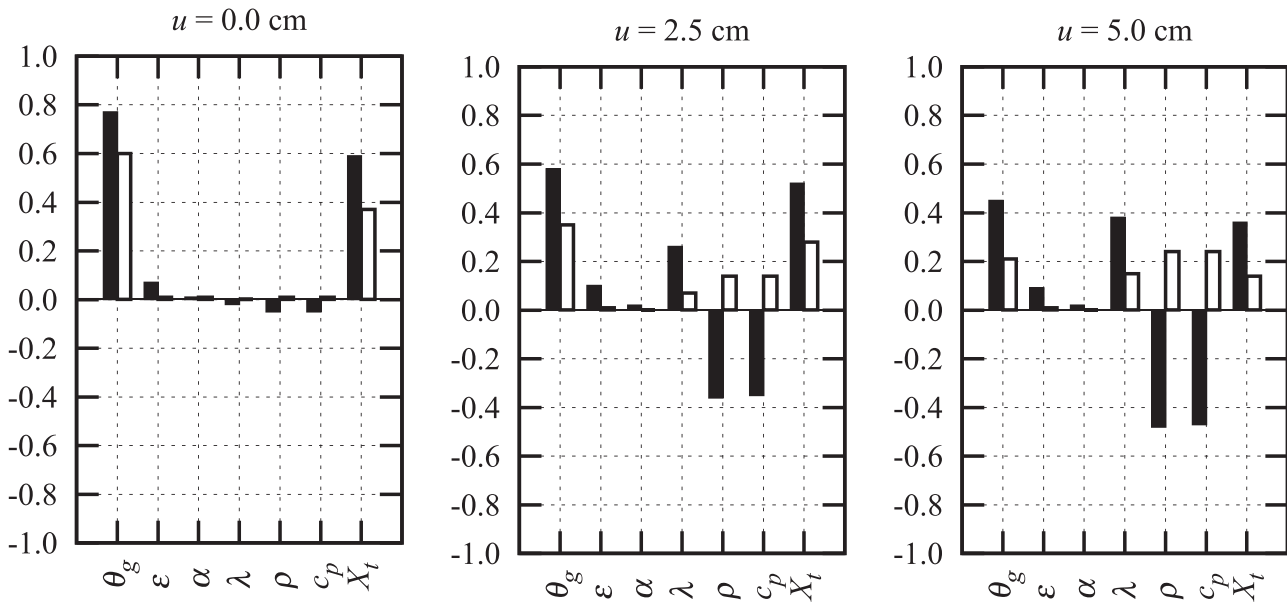
Fig. 2. Spearman's rank correlation coefficient (black) and first order Sobol indices (white) of the simulated wall.

of all results. It can be easily derived that $\sum S_i \leq 1$, where results $\sum S_i \approx 1$ indicate that there are no interactions between the variables.

The Sobol indices can be estimated by a Monte Carlo simulation [15]. In this paper, a conceptual implementation for the estimation of the Sobol index is used [14] with a slice size of 200. This method is also known as "method of slices".

3.2. Results of the sensitivity analysis

The results of the sensitivity analysis are displayed in Fig. 2. The variance of the calculated temperatures at the surface $u=0$ cm is controlled by the scatter of the gas temperature θ_g and the model uncertainty X_t: the sum of both first order Sobol indices is 0.98. Hence the influence of the scatter of all other variables on the results can be disregarded.

At the center $u=5.0$ cm of the examined wall, the physical properties become more influential. The calculated Spearman's rank correlation coefficients show, that the basic variables ρ and c_p have a negative correlation: increasing these values leads to lower temperatures. This is accordance with the differential Eq. (1). The sum of the first order Sobol indices $S_\lambda + S_\rho + S_{c_p}$ is 0.63, which means that 63 % of the variance of the calculated temperatures is controlled by the uncertainty of the thermal properties. The basic variables θ_g and X_t contribute 36 % to the variance of the calculated temperatures. The sensitivity of the basic variables α and ε is not mentionable.

The results for $u=2.5$ cm are between the previously discussed results, the effect of the scatter of α and ε on the results is also of inferior importance.

3.3. Conclusions of the sensitivity analysis

The calculated sensitivities reveal, that the basic variables α and ε can be fixed to their nominal values. The variance of the calculated temperatures at the surface is controlled by the uncertainty of the gas temperature and the model uncertainty. The accuracy of the calculated temperatures at the surface of the wall could only be reduced, if the variance of the basic variables θ_g and X_t would be reduced by improving the testing procedure or the accuracy of the physical model.

The sensitivity against the physical parameters increases with the distance from the surface u. Improving the mathematical formulation of the physical properties can lead to a reduction of the variance of the calculated temperatures.

4. Nonlinear optimization of the thermal conductivity

4.1. Applied methods

The laboratory tests with continuous slabs carried out by Kordina and Wesche [16] are used for the calibration problem, which is solved by means of nonlinear optimization. The plates have been heated on bottom and the influence of the layout of the upper reinforcement on the fire resistance has been studied. The temperatures at different distances u, as displayed in Fig. 3, have been recorded for plate number 2 up to the failure after 92 min. The height of the plate was 10 cm and a moisture content of 3.6 % has been measured at the date of test. The reported data contains a certain scatter, because the measured temperatures are from one thermocouple at each location only. Also thermocouples – not plate thermometers – have been used to determine the temperature in the furnace.

The difference between the calculated and measured temperatures $\Delta\theta = \theta_{cal} - \theta_{exp}$ is used to judge on the accuracy of the results. The mean

$$\Delta\bar{\theta} = \frac{1}{n}\sum_{i=1}^{n}(\theta_{cal,i} - \theta_{exp,i}) = \frac{1}{n}\sum_{i=1}^{n}\Delta\theta_i \tag{7}$$

and the variance

$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(\Delta\theta_i - \Delta\bar{\theta})^2 \tag{8}$$

are evaluated for all n measured temperatures. The thermal properties according to EN 1992-1-2 are considered as reference and the statistical key data $\Delta\bar{\theta}$ and s^2 are calculated. As second step, constant material properties – as proposed in ENV 1992-1-2 [17] for simple
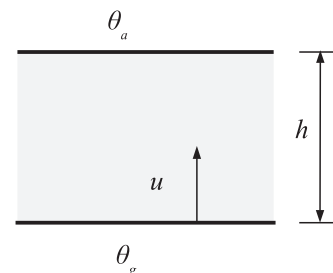

Fig. 3. Concrete slab, fire exposure at bottom.

Table 3
Thermal properties for recalculation of laboratory tests.

parameter	unit	value
heat transfer coefficient: α	[W/m^2 K]	exposed: 25
		unexposed: 4
emissivity: ε	[-]	0.7
density: ρ(20 °C)	[kg/m^3]	2400
moisture content: u	[%]	3
thermal conductivity: λ	[W/m K]	lower limit

calculations – are examined. Finally the method proposed by Nelder and Mead [18] is used for the nonlinear optimization of the thermal conductivity by minimizing the variance given by Eq. (8). It is a direct search algorithm, which does not need the local derivatives of the considered function.

4.2. Results according to EN 1992-1-2

The measured temperatures of plate 2 [16] are recalculated using the material properties of EN 1992-1-2. The corresponding parameters are given in Table 3 and the calculated and measured temperatures are displayed in Fig. 4. The mean difference of the calculated temperatures $\Delta\bar{\theta}$ is −13 K with a variance $s^2 = 883$ K^2. The temperatures at the fire exposed surface at u=1 cm are overestimated, where the temperatures at the unexposed surface (u = 7 cm) are underestimated by calculation. The biggest deviations occur at u=7 cm for the first 40 min of fire exposure: the calculated temperature are remarkable lower compared to the laboratory results.

4.3. Results for constant material properties

Constant material properties for simple calculations are given in ENV 1992-1-2. It is proposed to use a constant density $\rho = 2300$ kg/m^3, a constant specific heat $c_p = 1000$ J/kg K and constant thermal conductivity $\lambda = 1.6$ W/mK for siliceous (1.3 W/m K for calcareous) aggregates.

A constant density of $\rho = 2400$ kg/m^3 has been chosen in accordance with previously published results by Achenbach and Morgenthal [1]. The recommended value $c_p = 1000$ J/kg K has been adopted from ENV

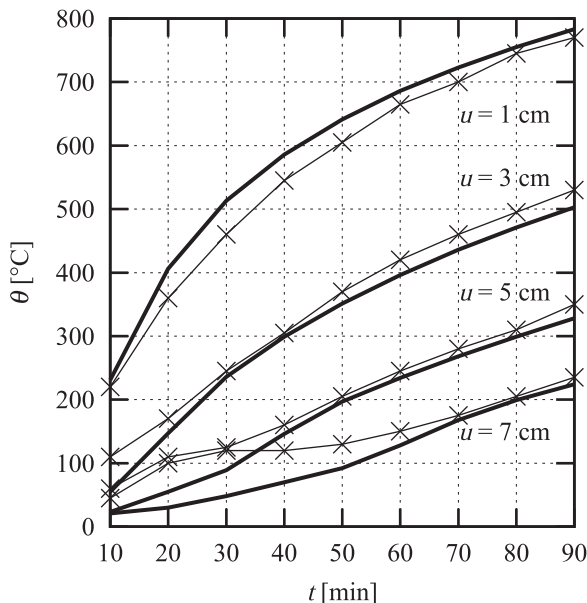**Fig. 4.** Calculated (━) and measured (×) temperatures of the slab in dependence from the distance to the surface u, material properties acc. to Table 3.

Table 4
Variance s^2 and mean $\Delta\bar{\theta}$ for $\theta_{cal} - \theta_{exp}$ for constant material properties.

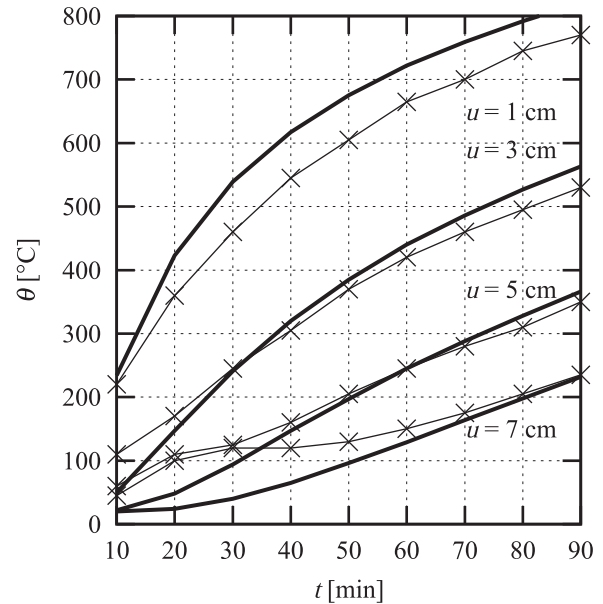| λ [W/m K]	s^2 [K^2]	$\Delta\bar{\theta}$ [K]
1.6	3078	66
1.3	2297	44
1.0	1830	15
0.9	1773	4
0.8	1780	−9

Fig. 5. Calculated (━) and measured (×) temperatures of the slab in dependence from the distance to the surface u with constant material properties: $\lambda = 0.9$ W/m K, $\rho = 2400$ kg/m^3, $c_p = 1000$ J/kg K.

1992-1-2. The boundary conditions at the surface α and ε are taken from Table 3 for the examinations. The variance s^2 and mean $\Delta\bar{\theta}$ in dependence from the thermal conductivity λ are given in Table 4: the least variance is calculated for $\lambda = 0.9$ W/m K. The calculated temperatures, using constant material properties, are shown in Fig. 5. Comparing the results of the simple material properties with the advanced material properties of EN 1992-1-2 shows, that the temperatures for u=5 and 7 cm differ only slightly. For u=3 cm, the measured temperatures are overestimated after 30 min of fire exposure. The largest deviations occur for u=1 cm, which shows clearly the limit of constant material properties.

4.4. Results for optimized material properties

The sensitivity analysis reveals, that the parameters α and ε can be fixed to their nominal values, which are given in Table 3. The scatter of the calculated temperatures within the cross section is controlled by the physical parameters λ, ρ and c_P. Though it is assumed in the performed sensitivity analysis, that these parameters are not correlated due to the lack of statistical key data, this can be doubted by practical considerations. The density ρ and the specific heat c_p are both related to the mass, so there must be at least a correlation between these both variables. Therefore it seems to be reasonable to fix both values and to use the thermal conductivity λ for fitting. This is also justified, because the heat flux on the surface is controlled by λ, as indicated by Eq. (2). The effect of this equation can also be seen in the results for constant material properties: the biggest deviations are observed close the fire exposed surface.

Therefore the thermal conductivity λ is considered in the nonlinear

Table 5
Nonlinear optimization of the thermal conductivity λ: initial and final values.

θ [°C]	$\lambda_{initial}$ [W/m K]	λ_{final} [W/m K]
0	2.0	2.44
200	1.6	1.30
400	1.2	1.00
≥600	1.0	0.70

optimization, while the density and specific heat are fixed to $\rho = 2400$ kg/m^3 and $c_p = 1000$ J/kg K. It is assumed, that λ is piecewise linear and determined by the values for $\theta = 0$, 200, 400 and 600 °C. The Nelder-Mead method [18] is used to find those values for λ, which minimize the variance s^2 given by Eq. (8). The additional condition $\Delta \bar{\theta} \geq 0$ K is introduced to guarantee that the mean temperatures are overestimated. Therefore the values of the upper limit of thermal conductivity according to EN 1992-1-2 are used as initial values to ensure, that the calculated temperatures are higher than the experimental results to allow more optimization cycles.

The initial and final values for λ are given in Table 5 and the final function is plotted in Fig. 7. The results of the calculated temperatures for the optimized function of λ is given in Fig. 6. The evaluation of $\Delta \theta = \theta_{cal} - \theta_{exp}$ leads to $\Delta \bar{\theta} = 1$ K and $s^2 = 584$ K^2, which means that the results are more accurate than those obtained with material properties from EN 1992-1-2. This is also visible in Fig. 6: there are small differences for u=5 cm and the results for u=7 cm are closer to the test results for the first 40 min of the test.

The shape of temperature dependent, simplified thermal conductivity is determined by the values at 0 °C, 200 °C and 600 °C, as indicated in Fig. 7. The value at 400 °C, obtained by nonlinear optimization, is not needed for the description of the piecewise linear function. The optimized values for $\theta \leq 100$ °C are higher than the upper limit of EN 1992-1-2, which leads to a faster heat transfer to the unexposed surface. It is also remarkable, that a constant value of $c_p = 1000$ J/kg K has been assumed in the optimization. Though the effect of evaporating water can be measured on small specimen and is considered in the material properties of EN 1992-1-2 by the variable $c_{p,h}$, it can be neglected in the optimization of λ.
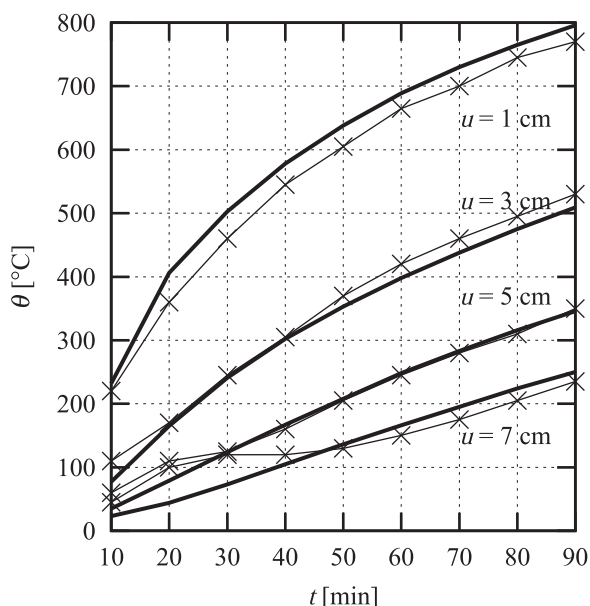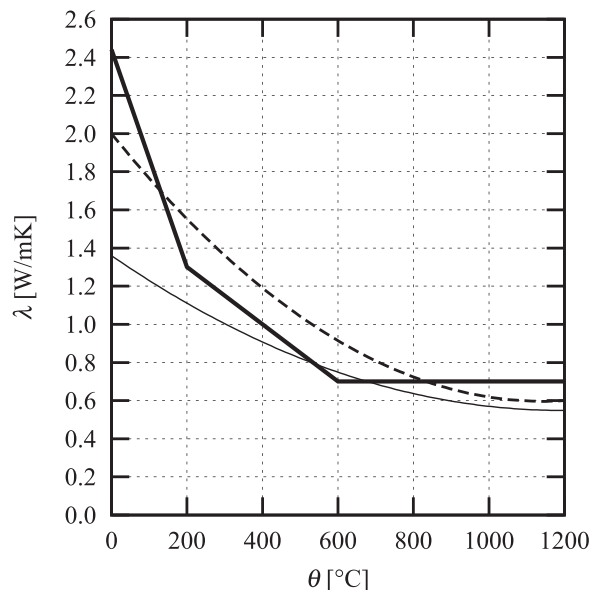

Fig. 7. Thermal conductivity λ of concrete: optimized function (━), lower (-) and upper (- - -) limit acc. to EN 1992-1-2.

5. Validation

5.1. Applied methods

The proposed optimized thermal properties (λ_{final} according to Table 5 and Fig. 7, $\rho = $ const $= 2400$ kg/m^3, $c_p = $ const $= 1000$ J/kg K) are checked by the recalculation of measured temperatures of concrete columns heated by a standard fire. The results are compared to the calculated temperatures using the advanced thermal properties of EN 1992-1-2, with the parameters described in Table 3.

The considered columns have been tested in Germany [19] and Sweden [20], the cross sections are displayed in Fig. 8. The columns from Germany had square cross section of $b = h$= 20 and 30 cm and have been heated on all surfaces. A moisture content from $u = 2.0$ to 6.0 % and siliceous aggregates are documented. The reported temperatures are mean values of different columns and thermocouples − the scatter of single measurements is smoothened. The temperature in the furnace has been measured with thermocouples. The temperatures for a distance to the surface of 0, 1, 3, 4, 6, 10 and 15 cm are calculated for the time steps $t = 10$, 20, …90 min. A total number of n=79 data points is evaluated.

The three columns tested in Sweden had a cross section of $b = h = 20$ cm and have been heated on three surfaces, as indicated in Fig. 8. A high moisture content of 6 % is reported. The measured temperatures are only documented in one figure with smoothened
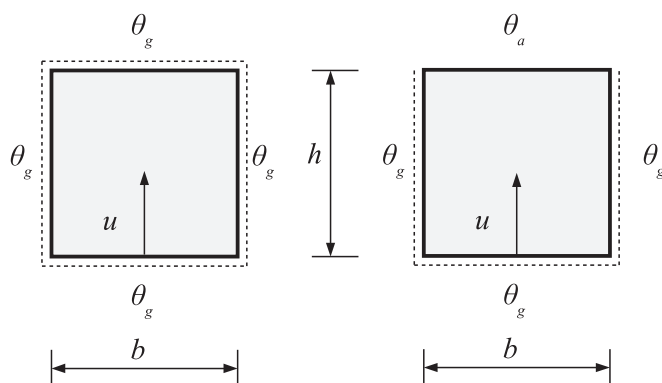

Fig. 6. Calculated (━) and measured (×) temperatures of the slab in dependence from the distance to the surface u with optimized thermal conductivity λ ($\rho = $ const $= 2400$ kg/m^3, $c_p = $ const $= 1000$ J/kg K).

Fig. 8. Tested concrete columns from Germany (left) and Sweden (right), heated surface indicated by (- - -).

Table 6
Statistical key data of the validation with columns using thermal properties of EN 1992-1-2 (EN) and optimized thermal conductivity λ (OPT) ($\rho = \text{const} = 2400 \text{ kg/m}^3$, $c_p = \text{const} = 1000 \text{ J/kg K}$), n=number of results.

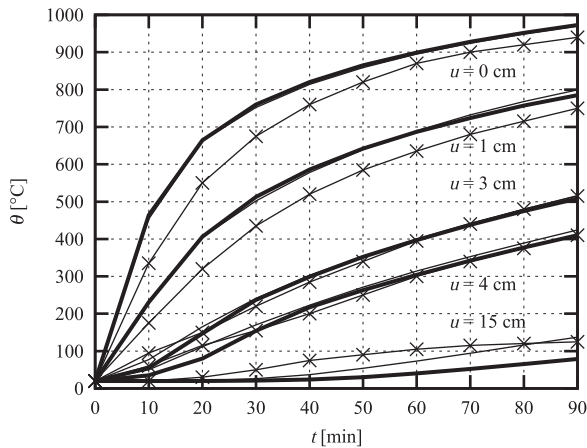| parameter	unit	Germany [19]		Sweden [20]	
		EN	OPT	EN	OPT
n	[-]	79	79	32	32
$\Delta\bar{\theta}$	[K]	5	21	39	59
s^2	[K^2]	1806	1171	2109	1479
s	[K]	42	34	46	38

Fig. 9. Measured ($\times$) and calculated temperatures of columns $b = h = 30$ cm from Germany [19] in dependence from the distance to the surface u with thermal properties acc. to EN 1992-1-2 (━) and with optimized thermal conductivity λ (-) ($\rho = \text{const} = 2400 \text{ kg/m}^3$, $c_p = \text{const} = 1000 \text{ J/kg K}$).
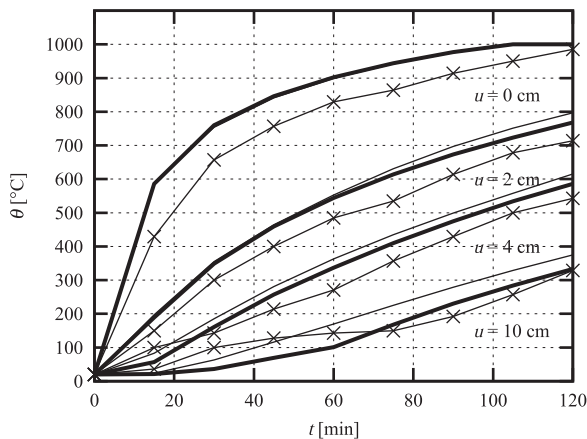

Fig. 10. Measured ($\times$) and calculated temperatures of columns from Sweden [20] in dependence from the distance to the surface u with thermal properties acc. to EN 1992-1-2 (━) and with optimized thermal conductivity λ (-) ($\rho = \text{const} = 2400 \text{ kg/m}^3$, $c_p = \text{const} = 1000 \text{ J/kg K}$).

curves, therefore only the time steps $t = 15, 30, \ldots 120$ min are considered for a distance to the surface of 0, 2, 4 and 10 cm. A number of $n = 32$ data points is calculated. It is not clear, if the documented temperatures are from one thermocouple of one column, or if they are averaged over all three tested columns. So the data must be interpreted carefully. But the data is assumed to be valuable, because of the high moisture content and long fire duration of 120 min.

5.2. Results

The statistical key data of the recalculation is documented in Table 6. The measured and calculated temperatures of the columns

$b = h = 30$ cm tested in Germany are displayed in Fig. 9 for a set of representable distances u. The results for both examined formulations of the thermal properties are close to each other for the distances $u = 0$ to 4 cm, only the results at the centroid for $u = 15$ cm differ. This is in accordance with the results of the sensitivity analysis: the calculated temperatures inside the cross section are sensitive against the thermal properties. The temperatures close to the surface are overestimated by calculation, which can also be observed by the recalculation of slabs in the previous section. The proposed simplified thermal properties enhance the prediction of the temperatures at the centroid, as indicated in Fig. 9. This influences also the statistical key data of the differences $\Delta\theta$: the mean value $\Delta\bar{\theta}$ is increasing and the variance s^2 is reduced.

The results of the recalculation of the columns from Sweden are displayed in Fig. 10. Though a large scatter of the results is visible, the conclusions of the tests from Germany are also valid. Both formulations of the thermal properties are close to each other at the surface and differ at the centroid. The proposed simplifications mainly affect the temperatures at the centroid and lead to a reduction of the variance s^2, as indicated in Table 6.

6. Conclusions

The sensitivity analysis reveals, that calculated temperatures at the surface are highly sensitive against the uncertainty of the gas temperature and the model uncertainty. Both uncertainties can be hardly reduced. The influence of the uncertainty of the material properties increases with the distance to the surface. Hence the uncertainty of the calculated temperatures can be reduced by improved material properties.

It can be shown, that it is possible to fix the density ρ and the specific heat c_P to constant values and to consider only the thermal conductivity λ as temperature dependent. The applied methods leads to a simple, piecewise linear function for λ. The proposed function is for small temperatures out of the range, which is specified by the lower and upper limit given in EN 1992-1-2. This needs further discussion, especially a bigger database of laboratory tests for optimization and validation should be considered.

Though the obtained results of the calibration and validation are encouraging for further simplifications , it is not clear, if the proposed values are valid for all dimensions and types of concrete cross sections like beams and hollow core slabs. It must be pointed out that only one slab with a thickness of 10 cm has been used for optimization and that the results of the validation are limited to tests on columns from two laboratories. Therefore the proposed methodology should be applied to a bigger number of laboratory tests. Also the effect on the fire resistance and the mechanical behavior of the considered member needs further investigation.

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