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CONSTRAINED TRAFFIC DEMAND MODELS - SIMULTANEOUS DISTRIBUTION AND MODE CHOICE

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Abstract. *Unconstrained models are very often found in the broad spectrum of different theories of traffic demand models. In these models there are none or only one-sided restrictions influencing the choice of the individual. However in the traffic demand different deciding dependencies of the traffic volume with regard to the specific conditions of the territory structure potentials exist. Kichhoff and Lohse introduced bi- and tri-linearly constrained models to show these dependencies. In principle, the dependencies are described as hard, elastic and open boundary sum criteria. In this article a model is formulated which gets away from these predefined boundary sum criteria and allows a free determination of minimal and maximal boundary sum criteria. The iterative solution algorithm is shown according to a FURNESS procedure at the same time. With the approach of freely selectable minimal and maximal boundary sum criteria the modeling transport planner gets the possibility to show the traffic event even better. Furthermore all common boundary sum criteria can be calculated with this model. Therewith the often necessary and sensible standard and special cases can also be modeled.*

1 INTRODUCTION

Traffic demand can be described as satisfaction of the need “locomotion”. This is caused by the spatial separation of different activities (e.g.: home, work etc.). Corresponding to this traffic demand models shall qualitatively and quantitatively describe the translocations realized according to the causes of the traffic. This means trip generation, distribution, mode choice and route choice under political, economic and traffic planning conditions.

The models used for the calculation of traffic demand offer a broad spectrum of most different model theories. Nevertheless demand models almost exclusively used in the strategic transport planning are differentiated by spatial elements. Traffic analysis zones are origins and destinations of the traffic demand. They therefore require the analysis of traffic infrastructural uses. In high-quality models destination choice and mode choice (partly also route choice) are calculated simultaneously[1] for every single demand segment (e.g.: Origin-Destination-Groups). On the one hand, the simultaneity justifies itself because the destination choice and the mode choice in many cases are interdependent. On the other hand, the sequential model structures show model conditional deficiencies.

Benefit maximizing approaches are frequently used in practice. These very often are unconstrained. A model, which only is subject to a restriction on the side of the origin is also an unconstrained model (e. g.: on the side of origin fixed Logit-Model). A constraint only results from a restriction on the side of the origin and on the side of destination. Therefore these model theories assume that none or only one sided restrictions influence the choice of the individuals. So the absence of constraints (also called side conditions or boundary sum criteria, the criteria refer to the boundary sums of the trip flow matrix) allows the benefit maximization of the traffic participants in the open decision space. But this does not correspond to the real traffic situation! Different decisive dependencies of traffic volume and the specific conditions of the territory structure potentials exist in the actual traffic situation (e. g.: number of accommodations, number of workplaces, size of shopping centers etc.).

2 BOUNDARY SUM CRITERIA

However, to show these different dependencies of the traffic volume with regard to different specific conditions, the mentioned boundary sum criteria must certainly enter the model theory. Corresponding to this the boundary sum criteria are complied in the doubly constrained model of destination choice of Kirchhoff (cp. [4]).

$$\begin{aligned} v_{ij} &= BW_{ij} \cdot fq_i \cdot fz_j \\ \sum_j v_{ij} &= Q_i \\ \sum_i v_{ij} &= Z_j \end{aligned} \tag{1}$$

[1] Whether it is a (pure) simultaneous model or (hierarchical) Nested-Logit-Model is not decisively. In these models only the scaling or the nest parameter is frequently different.

On this basis Lohse (cp. [6]) developed a three sided constrained model of destination choice and mode choice.

$$v_{ijk} = BW_{ijk} \cdot fq_i \cdot fz_j \cdot fa_k$$

$$\sum_j \sum_k v_{ijk} = Q_i$$

$$\sum_i \sum_k v_{ijk} = Z_j \qquad (2)$$

$$\sum_i \sum_j v_{ijk} = A_k$$

Q	origin traffic volume
Z	destination traffic volume
A	traffic volume of a mode (Modal Split)
i	Index of origin ($i = 1,...,m$)
j	Index of destination ($j = 1,...,n$)
k	Index of mode ($k = 1,...,l$)
v	traffic flow

These models with boundary sum criteria have to be assigned to the econometric models. As choice-theoretical models of the discrete choice these assume that a person has a choice of a set of alternatives and this person tries to maximize the benefit through his choice.

Besides the dependency of the traffic volume on the territory structure potentials further dependencies of the accessibility exist in the actual traffic situation. These must also be paid attention to. Specific restrictive conditions concerning the traffic participants competing for the potentials, however, have to be taken into account too. Authoritative territory structure sizes, accessibility as well as specific conditions of the traffic situation in the investigation area influence the actual traffic situation basically and therefore have to be included in traffic planning calculation procedures. Furthermore it has to be taken into account that depending on the activity the following different dependencies of the traffic volume on these influence sizes exists:

- exclusively on the territory structure sizes
- on the territory structure sizes and the accessibility
- exclusively on the accessibility

In principle, these dependencies can be divided up into hard, elastic and open boundary sum criteria.

2.1 Hard boundary sum criteria

Hard boundary sum criteria consider the spatially not substitutable compulsory activities[2] (e. g.: work, education). So the destination cannot be changed immediately. For example, normally it is not possible to change the place of work at short notice. The expected value of the corresponding volume of traffic therefore is exclusively calculated with the authoritative structure sizes. The accessibility does not play a role for the compliance with the boundary sum. This way a boundary sum criterion results that the traffic volume has to comply with in the later calculation of the distribution and mode choice. The mathematical formulation of the hard boundary sum criteria in the 3 dimensional model (origin, destination, mode) therefore is:

[2] Destinations which have to be found and not (permanently) chosen or changed newly (e.g.: work, school) are described as not substitutable compulsory destinations. This is not the case at substitutable destinations. Here a new destination can be found (e.g: shopping) in dependence of the premises (permanently).

$$\sum_j \sum_k v_{ijk} = Q_i$$

$$\sum_i \sum_k v_{ijk} = Z_j \qquad (3)$$

$$\sum_i \sum_j v_{ijk} = A_k$$

Of course these hard boundary sum criteria by Lohse can also be transferred to the model by Kirchhoff. Lohse enhanced these boundary sum criteria to elastic boundary sum criteria (cp. Lohse et al. [6], Schiller [8]).

2.2 Elastic boundary sum criteria

Unlike hard boundary sum criteria in the context of substitutable activities (e. g.: shopping, other) the expected value of the volume of traffic results no longer exclusively from the authoritative structure sizes. Besides that the accessibility also plays a decisive role in the choice of competitive activities. So traffic participants also estimate the translocation effort in their destination choice for different shopping centers, for example. If the shopping center reachable with the lowest effort is not yet occupied totally by "competitive" traffic participants, the traffic participants will select this shopping center with a greater probability. They change their destination choice as soon as the destination reaches overload. So the trip generation only calculates maximum potentials (capacities) which do not have to be fully utilised, though. The traffic flows are calculated proportionally to the potentials and the accessibility in the distribution and mode choice. However, it is necessary not to exceed the potentials of the traffic analysis zones. This also means that the potentials are not made full use of, as long as the competition of the supply potentials and their distribution in the area do not force them to. It is a general precondition with elastic boundary sum criteria, though, that a sufficient capacity must be available in the traffic analysis zones to offer a compensation in another place. The mathematical formulation of the elastic boundary sum criteria in the 3 dimensional model therefore is:

$$\sum_j \sum_k v_{ijk} \leq Q_i^{max}$$

$$\sum_i \sum_k v_{ijk} \leq Z_j^{max} \qquad (4)$$

$$\sum_i \sum_j v_{ijk} = A_k$$

Q_i^{max} max. origin traffic volume
Z_j^{max} max. destination traffic volume

2.3 Open boundary sum criteria

In this (special-) case the traffic volume of substitutable activities results from accessibility of competitive destinations and the way of the authoritative structure sizes, whether a destination is available at all or not. In this case potential limits or capacity limits of the structure sizes do not have any effect. This case appears in traffic planning application cases rather seldomly, though. The mathematical formulation of the open boundary sum criteria in the 3 dimensional model would therefore appear as follows:

$$\sum_j \sum_k v_{ijk} \le Q_i^{\infty}$$

$$\sum_i \sum_k v_{ijk} \le Z_j^{\infty} \quad (5)$$

$$\sum_i \sum_j v_{ijk} = A_k$$

Thus combinations of boundary sum criteria arise depending on the examination object or on the considered demand segments, which can appear in numerous forms (cp. Table 1). If no open boundary sum criteria exists, the model is described as constrained.

	origin traffic volume (Q)	destination traffic volume (Z)	complete traffic volume	model constraints
1	hard	hard	fix	constrained
2	hard	elastic	fix	constrained
3	hard	open	fix	not constrained
4	elastic	elastic	fix	constrained
5	open	open	fix	not constrained

Table 1: Possibilities of the combination of boundary sum criteria

Caused by the fact that the boundary sum criteria must be adhered, a restrictive requirement is added to the benefit maximization unlike models without boundary sum criteria. The traffic participants still strive for the benefit maximum, but they can not achieve it in the same extent any more because of the limiting boundary sum criteria. So the traffic participants try to get close to the benefit maximum if possible and to minimize the arising loss of benefit. This means that because of the competing decision behaviors of the traffic participants, the matrix in the event space with boundary sum criteria differs least possible from the matrix in the event space without boundary sum criteria.

3 INFORMATION PROFIT MINIMIZATION

The assessment and decision process of the traffic participants represents a minimization of the benefit loss. Through this, that the traffic participants can not achieve the benefit maximum because of the limiting boundary sum criteria in the same extent any more and nevertheless try to get close to the benefit maximum if possible or try to minimize the arising benefit loss. This benefit loss can theoretically be modeled with the minimization of an interspace measure defined between two states (without and with boundary sum criteria). With the information profit minimization

$$I = -\sum \left[\alpha \cdot ln \left(\frac{\alpha}{\beta} \right) \right] \quad (6)$$

the information theory provides an adequate method to keep the benefit loss least possible. The tri-linear model with boundary sum criteria can also be described as an extreme value model with side conditions, in which the minimization of the information profit from the given assessment matrix to the sought-after traffic flow matrix is the object function.

$$I = \sum_i \sum_j \sum_k \left[v_{ijk} \cdot ln \left(\frac{v_{ijk}}{BW_{ijk}} \right) \right] \quad (7)$$

To calculate the minimization of the information profit, different optimization algorithms can be used. These are introduced in detail in Kirchhoff (cp. [4]), Lohse et al. (cp. [6]) and Schiller (cp. [8]).

Together with the assessment probabilities[3], the basically introduced boundary sum criteria and the mentioned optimization conditions concerning the information profit minimization, solution regulations and iterative procedures (e. g.: DETROIT, FRATAR, FURNESS, MULTI) can be described for a successive approximate solution of the tri-linear general model and its boundary sum criteria (cp. z. B.: [5], [6], [8]).

4 FURTHER ANALYSIS OF THE BOUNDARY SUM CRITERIA

Elastic boundary sum criteria are characterized by maximum potentials or maximum boundary sum criteria. These do not have to reach capacity, but may not be exceed it either. These maximum criteria were justified with the availabilities and restrictions of the real traffic situation. But there also are single (selective) destinations which always attract a minimum of traffic despite a bad accessibility. Even though these destinations show the same supply parameters as well as other destinations from the theoretical transport planning view. So these destinations have an attraction potential which can not or only very difficult be quantified. (Of course this minimal traffic volume can also increase up to a maximum which is dependent on the competition with other destinations.) This is only one of several reasons to generate minimal boundary sum criteria too. According to this, these minimal and maximal boundary sum criteria must be appropriately freely selectable, to be able to show the traffic situation as exactly as possible or necessary.

$$\begin{aligned}
\sum_j \sum_k v_{ijk} &\geq Q_i^{min} \\
\sum_j \sum_k v_{ijk} &\leq Q_i^{max} \\
\sum_i \sum_k v_{ijk} &\geq Z_j^{min} \\
\sum_i \sum_k v_{ijk} &\leq Z_j^{max}
\end{aligned} \tag{8}$$

Furthermore the choice of the boundary sum criteria should not limit the possibilities of the modeling transport planner and also allow the often necessary and sensible standard and special cases besides the general. One of these standard or special cases is to describe the boundary sum criteria as "hard".

$$\begin{aligned}
\sum_j \sum_k v_{ijk} &= Q_i \\
\sum_i \sum_k v_{ijk} &= Z_j
\end{aligned} \tag{9}$$

[3] The explicit analysis of determination of the assessment probability shall be renounced here. The combination of the efforts (keyword: Generalized Costs) and the real assessment function form (keyword: e-function or EVA-, Box Cox-, Box-Tuckey-fct. etc.) should always be taken into account.

Related to the approach of the minimal and maximal boundary sum criteria in fact the case occurs that the minimal boundary sum criteria are equal to the maximum boundary sum criteria.

$$\sum_j \sum_k v_{ijk} = Q_i^{min} = Q_i^{max}$$
$$\sum_i \sum_k v_{ijk} = Z_j^{min} = Z_j^{max} \tag{10}$$

The elastic boundary sum criteria

$$\sum_j \sum_k v_{ijk} \leq Q_i^{max}$$
$$\sum_i \sum_k v_{ijk} \leq Z_j^{max} \tag{11}$$

as well as the open boundary sum criteria

$$\sum_j \sum_k v_{ijk} \leq Q_i^{\infty}$$
$$\sum_i \sum_k v_{ijk} \leq Z_j^{\infty} \tag{12}$$

also represent a special case. It can be assumed that the not freely selectable minimal values of Lohse et al. are zero. The boundary sum criteria consequently have to be defined as follows relating to the approach of the freely selectable minimal and maximal boundary sum criteria:

$$\sum_j \sum_k v_{ijk} \geq Q_i^{min} = 0$$
$$\sum_j \sum_k v_{ijk} \leq Q_i^{max}$$
$$\sum_i \sum_k v_{ijk} \geq Z_j^{min} = 0$$
$$\sum_i \sum_k v_{ijk} \leq Z_j^{max} \tag{13}$$

and accordingly

$$\sum_j \sum_k v_{ijk} \geq Q_i^{min} = 0$$
$$\sum_j \sum_k v_{ijk} \leq Q_i^{\infty}$$
$$\sum_i \sum_k v_{ijk} \geq Z_j^{min} = 0$$
$$\sum_i \sum_k v_{ijk} \leq Z_j^{\infty} \tag{14}$$

The necessity or real existence of minimal boundary sum criteria is described in the preceding text, but no calculation approach for freely selectable minimal and maximal boundary sum criteria exists to date. The developed approach shall be shown in greater detail in the following due to its complexity.

5 SOLUTION METHODS WITH MINIMAL AND MAXIMAL BOUNDARY SUM CRITERIA

For the simplification and clarity hard boundary sum criteria only are considered on the side of origin. This means Q_i and not $Q_i^{min} = Q_i^{max}$! However minimal and maximal boundary sum criteria are considered on the destination side. Furthermore fixed mode quotas A_k are used. Herewith a model in the short notation of a tri-linear model

$$\begin{aligned} & v_{ijk} = BW_{ijk} \cdot fq_i \cdot fz_j \cdot fa_k \\ & \left.\begin{aligned} & \sum_j \sum_k v_{ijk} = Q_i \\ & \sum_i \sum_k v_{ijk} \geq Z_j^{min} \\ & \sum_i \sum_k v_{ijk} \leq Z_j^{max} \\ & \sum_i \sum_j v_{ijk} = A_k \end{aligned}\right\} boundary\ sum\ criteria \end{aligned} \tag{15}$$

can be formulated. Since this task is obviously not solvable the solution with the smallest information profit from all possible solutions has to be chosen. The convex optimization problem has to be enlarged correspondingly to Bregmann (cp. [1]).

$$I = \sum_i \sum_j \sum_k \left[v_{ijk} \cdot ln\left(\frac{v_{ijk}}{BW_{ijk}} \right) \right] \rightarrow Min! \tag{16}$$

The following affine linear side criteria have to be applied:

$$\begin{aligned} & \sum_j \sum_k v_{ijk} = Q_i \\ & \sum_i \sum_k v_{ijk} \geq Z_j^{min} \\ & \sum_i \sum_k v_{ijk} \leq Z_j^{max} \\ & \sum_i \sum_j v_{ijk} = A_k \end{aligned} \tag{17}$$

In turn this convex optimization problem (16) with the side criteria (17) can be characterized as a generalization of the information profit minimization. For the solution of this problem the saddle point of the accompanying LAGRANGE-equation

$$
\begin{aligned}
\Phi = &\sum_i \sum_j \sum_k \left[v_{ijk} \cdot ln\left(\frac{v_{ijk}}{BW_{ijk}} \right) - v_{ijk} \right] + \sum_i \lambda_i \cdot \left(\sum_j \sum_k v_{ijk} - Q_i \right) \\
&+ \sum_j \mu_j \cdot \left(Z_j^{min} - \sum_i \sum_k v_{ijk} \right) + \sum_j \nu_j \cdot \left(\sum_i \sum_k v_{ijk} - Z_j^{max} \right) \\
&+ \sum_k o_k \cdot \left(\sum_i \sum_j v_{ijk} - A_k \right)
\end{aligned} \tag{18}
$$

has to be determined (cp. Ellinger et al. [2]). The calculation of the minimum is treated approximately analogously for the calculation of the maximum. This means that own additional factors have to be included in the Lagrange-function. These factors have an sign restriction, though, so the complementarity condition and the not negativity condition must be extended by these inequalities. The quality of the saddle point can be described with the authoritative Kuhn-Tucker-conditions.

$$
\begin{aligned}
&ln\left(\frac{v_{ijk}}{BW_{ijk}} \right) + \left(\lambda_i + (\upsilon_j - \mu_j) + o_k \right) = 0 \\
&\sum_j \sum_k v_{ijk} - Q_i = 0 \\
&Z_j^{min} - \sum_i \sum_k v_{ijk} \leq 0 \\
&\sum_i \sum_k v_{ijk} - Z_j^{max} \leq 0 \\
&\sum_i \sum_j v_{ijk} - A_k = 0 \\
&\left. \begin{aligned} &\mu_j \geq 0 \\ &\upsilon_j \geq 0 \end{aligned} \right\} not\ negativity\ condition \\
&\left. \begin{aligned} &\sum_j \mu_j \cdot \left(Z_j^{min} - \sum_i \sum_k v_{ijk} \right) = 0 \\ &\sum_j \upsilon_j \cdot \left(\sum_i \sum_k v_{ijk} - Z_j^{max} \right) = 0 \end{aligned} \right\} complementarity\ condition
\end{aligned} \tag{19}
$$

The relation

$$
ln\left(\frac{v_{ijk}}{BW_{ijk}} \right) + \left(\lambda_i + (\upsilon_j - \mu_j) + o_k \right) = 0 \tag{20}
$$

corresponds with

$$
\begin{aligned}
v_{ijk} &= BW_{ijk} \cdot exp\left(\lambda_i + (\upsilon_j - \mu_j) + o_k \right) \\
&= BW_{ijk} \cdot fq_i \cdot fz_j \cdot fa_k
\end{aligned} \tag{21}
$$

By using (21) in (18) under consideration of (19) the following equivalent minimum problem arises in which only known sizes and the parameters may stand.

$$\begin{aligned}\psi = &\sum_i \sum_j \sum_k BW_{ijk} \cdot exp\left(-\left(\lambda_i + (\upsilon_j - \mu_j) + o_k\right)\right) \\ &+ \sum_i \lambda_i \cdot Q_i - \sum_j \mu_j \cdot Z_j^{min} + \sum_j \upsilon_j \cdot Z_j^{max} + \sum_k o_k \cdot A_k \rightarrow Min!\end{aligned} \tag{22}$$

The partial derivations of the zero point consideration have to be formed for the dual object function. The Lagrange-function is going to reduce at a part of its domain and to maximize the restricted function. This restricted function only depends on the Lagrange -factors (dual variables). The only side criteria of the dual problem are the sign restrictions for the variables which are part of the inequalities of the primal problem.

$$\begin{aligned}\frac{\partial \psi}{\partial \lambda_i} &= -exp(-\lambda_i) \cdot \sum_j \sum_k BW_{ijk} \cdot exp\left(-\left((\upsilon_j - \mu_j) + o_k\right)\right) + Q_i = 0 \\ \frac{\partial \psi}{\partial \mu_j} &= exp(\mu_j) \cdot \sum_i \sum_k BW_{ijk} \cdot exp\left(-\left(\lambda_i + \upsilon_j + o_k\right)\right) - Z_j^{min} = 0 \\ \frac{\partial \psi}{\partial \upsilon_j} &= -exp(-\upsilon_j) \cdot \sum_i \sum_k BW_{ijk} \cdot exp\left(-\left(\lambda_i - \mu_j + o_k\right)\right) + Z_j^{max} = 0 \\ \frac{\partial \psi}{\partial o_k} &= -exp(-o_k) \cdot \sum_i \sum_j BW_{ijk} \cdot exp\left(-\left(\lambda_i + (\upsilon_j - \mu_j)\right)\right) + A_k = 0\end{aligned} \tag{23}$$

In this place a simple coordinate descent method for Q_i and A_k can be derived for the solution of the dual problem. Is for $\lambda_i(p)$ in the p-th iteration step an approximation for the dual variables known and consequently for $(-\lambda_i)$ the corresponding approximation factor fq_i, arises of the way down in the λ_i-direction in the $(p+1)$-th iteration step as new approximation the clear solution of the equation:

$$-exp\left(-\lambda_i(p+1)\right) \cdot \sum_j \sum_k BW_{ijk} \cdot exp\left(-\left((\upsilon_j - \mu_j) + o_k\right)\right) + Q_i = 0 \tag{24}$$

Now the approach shown in equation (24) of a coordinate descent method can easily be transferred into a **FURNESS**-procedure, as the most popular iterative procedure[4]. But before this approach shall further be described, the common names have to be changed. The factors in the approach (e. g.: fz_j) and in the approximations of the iterative procedure (e. g.: $fz_j(p)$) should not correspond to each other in name. For example $\prod_p fz_j(p)$ is only an approximation to fz_j, so $\prod_p fz_j(p) \cong fz_j$. So the factors in the approach will be furthermore described as fz_j but the factors in the iterative procedure as $gz_j(p)$, so $\prod_p gz_j(p) \cong fz_j$.

[4] The **MULTI**-procedure calculates all dimensions simultaneously and is consequently faster than the FURNESS-procedure. This iterative solution method was also adapted to minimal and maximum boundary sum criteria. The procedure shall not be further deepened in this place, though.

A modified procedure of the already mentioned FURNESS-procedure can be derived from equation (24):

$$exp\left(-\lambda_i\left(p+1\right)\right)=\frac{Q_i}{\sum_j\sum_k BW_{ijk}\cdot exp\left(-\left(\left(\upsilon_j-\mu_j\right)+o_k\right)\right)}$$
$$gq_i\left(p+1\right)=\frac{Q_i}{\sum_j\sum_k BW_{ijk}\cdot gz_j\left(p\right)\cdot ga_k\left(p\right)} \tag{25}$$

At the following way down in μ_i- and ν_j-direction the not negativity conditions of (19) are taken into account. The new approximations of the equations

$$-exp\left(\mu_j\right)\cdot\sum_i\sum_k BW_{ijk}\cdot exp\left(-\left(\lambda_i+\upsilon_j+o_k\right)\right)-Z_j^{min}=0$$
$$-exp\left(-\upsilon_j\right)\cdot\sum_i\sum_k BW_{ijk}\cdot exp\left(-\left(\lambda_i-\mu_j+o_k\right)\right)+Z_j^{max}=0 \tag{26}$$

only then are certain if they lie in the permitted area of $\mu_j\geq 0$ or $\upsilon_j\geq 0$. If this is not the case, so μ_i =0 or ν_j =0.

Considering the minimal and maximal boundary sum criteria this means for the new *(p+1)*-approximation of gz_j:

$$gz_j\left(p+1\right)=max\left\{1,\frac{Z_j^{min}}{\sum_i\sum_j BW_{ijk}\cdot gq_i\left(p+1\right)\cdot ga_k\left(p\right)}\right\}$$
$$\cdot min\left\{1,\frac{Z_j^{max}}{\sum_i\sum_j BW_{ijk}\cdot gq_i\left(p+1\right)\cdot ga_k\left(p\right)}\right\} \tag{27}$$

If the traffic volume $Z_j(p)$ exceeds the minimal or maximal restrictions Z_j^{min} or Z_j^{max}, the algorithm corrects $gz_j(p+1)$ correspondingly. The factor $gz_j(p+1)=1$ arises if $Z_j(p)$ lies within the minimal or maximal restrictions according to the information profit minimization where no further information shall be introduced in the matrix. This is realized with the factor 1. According to the gq_i-factor the ga_k-factor can also be derived.

$$ga_k\left(p+1\right)=\frac{A_k}{\sum_i\sum_j BW_{ijk}\cdot gq_i\left(p+1\right)\cdot gz_j\left(p+1\right)} \tag{28}$$

Start solution: $gq_i\left(1\right)=gz_j\left(1\right)=ga_k\left(1\right)=1;\ \ v_{ijk}\left(1\right)=BW_{ijk}$

So with this explanatory model it is not only possibly to choose minimal and maximal boundary sum criteria freely and consequently display the (traffic-) infrastructure relating to the conditions in a better way. Different boundary sum criteria on the side of the production and/or attraction within one demand segment can also be calculated. To this an example calculation is shown in Figure 1 where different combinations of boundary sum criteria are calculated mixedly. The TAN 1 lies very favorable, however the TAN 5 lies very unfavorable.

Corresponding the assessment probability BW_{ijk} is very high for the TAN 1, turned over correspondingly at TAN 5. This example shows the contrasts of the boundary sum criteria and the accessibility or assessment and the effects of the minimal and maximal boundary sum criteria very good.

Multi-Procedure:

i / j PrT	1	2	3	4	5		Qi-actual
1	0,00	6,03	6,03	5,25	22,26		39,56
2	2,68	0,00	39,35	33,61	0,00		75,64
3	2,68	39,35	0,00	34,12	0,00		76,15
4	2,56	36,94	37,50	0,00	0,00		77,00
5	31,65	0,00	0,00	0,00	0,00		31,65
							300,00
Zj-actual	39,56	82,32	82,88	72,98	22,26	**300,00**	**300,00**

i / j PuT	1	2	3	4	5		Qi-actual
1	0,00	2,82	2,82	2,45	10,40		18,48
2	1,25	0,00	17,29	11,17	0,00		29,71
3	1,25	17,30	0,00	14,29	0,00		32,84
4	1,20	12,28	15,70	0,00	0,00		29,18
5	14,78	0,00	0,00	0,00	0,00		14,79
							125,00
Zj-actual	18,48	32,39	35,81	27,91	10,40	**125,00**	**125,00**

i / j B	1	2	3	4	5		Qi-actual
1	0,00	0,73	0,73	0,64	2,71		4,82
2	0,33	4,83	4,83	4,21	17,85		32,05
3	0,33	3,63	0,00	2,28	0,00		6,24
4	0,31	0,22	2,51	0,00	0,00		3,04
5	3,86	0,00	0,00	0,00	0,00		3,86
							50,00
Zj-actual	4,82	9,42	8,07	7,13	20,56	**50,00**	**50,00**

i / j P	1	2	3	4	5		Qi-actual
1	0,00	1,85	1,85	1,61	6,83		12,13
2	0,82	0,00	0,36	0,00	0,00		1,19
3	0,82	0,00	0,00	0,37	0,00		1,19
4	0,79	0,00	0,00	0,00	0,00		0,79
5	9,71	0,00	0,00	0,00	0,00		9,71
							25,00
Zj-actual	12,13	1,85	2,21	1,98	6,83	**25,00**	**25,00**

Comparison sums:

Qi	75,00	138,58	116,42	110,00	60,00	**500,00**
bsc min	0,00	100,00	100,00	110,00	60,00	**370,00**
bsc max	75,00	150,00	150,00	110,00	150,00	**635,00**
Zj	75,00	125,98	128,97	110,00	60,05	**500,00**
bsc min	0,00	100,00	100,00	110,00	60,00	**370,00**
bsc max	75,00	150,00	150,00	110,00	150,00	**635,00**

Figure 1: Traffic flow matrixes with minimal and maximal boundary sum criteria

Mode	Modal-Split	reference	actual
Private Transport (PrT)	0,60	300,00	300,00
Public Transport (PuT)	0,25	125,00	125,00
Bicycle (B)	0,10	50,00	50,00
Pedestrian (P)	0,05	25,00	25,00
aggregate	1,00	500,00	500,00

Figure 2: General parameters of Figure 1

6 OBSERVANCE OF POTENTIALS WITHIN THE SOLUTION METHODS

The derivation of the solution method should not be complicated unnecessarily, so the observance of the potentials should be carried out only in this place. The ability of a traffic analysis zone to take a (origin- or destination-) traffic volume up to a maximum potential limit is indicated as potential. At example of the destination traffic potential meant this $Z_j^P \leq Z_j^{max}$. Just at procedures where minimal and maximal boundary sum criteria do not agree with the potentials, the single structure sizes must also have influence on the assessment probability. This justifies itself by the fact that greater potentials are probability theoretically knowner and perform consequently a greater appeal than smaller potentials. These sizes (publicity by potential of the destination) consequently get involved in the calculation independently of the direct assessment probability (effort). The inclusion of the potentials is as a rule carried out in the first iteration step and also represents a part of the start solution.

$$v_{ijk}(1) = BW_{ijk} \cdot Z_j^P \tag{29}$$

At this exemplary start solution the destination potentials therefore have influence on the calculation of traffic flows (together with the assessment probability) of the first iteration step. The start solution can naturally contain only such factors which are also confessed at the beginning of the calculation, though. An explanatory model at the calculation of traffic flows therefore can be written down in the following form:

$$\begin{aligned} v_{ijk} &= BW_{ijk} \cdot \frac{Q_i}{V} \cdot q_i \cdot \frac{Z_j^P}{V^P} \cdot z_j \cdot \frac{A_k}{V} \cdot a_k \cdot V \cdot c \\ &= BW_{ijk} \cdot \frac{Q_i \cdot A_k}{V} \cdot \frac{Z_j^P}{V^P} \cdot q_i \cdot z_j \cdot a_k \cdot c \end{aligned} \tag{30}$$

Since the factors q_i, z_j and a_k have the value 1 in the first iteration step, the start solution

$$v_{ijk}(1) = BW_{ijk} \cdot \frac{Q_i}{V} \cdot \frac{Z_j^P}{V^P} \cdot \frac{A_k}{V} \cdot V \cdot c \tag{31}$$

for equation (16) can be formed as follows:

$$I = \sum_i \sum_j \sum_k \left[v_{ijk} \cdot ln \left(\frac{BW_{ijk} \cdot \frac{Q_i \cdot A_k}{V} \cdot \frac{Z_j^P}{V^P} \cdot q_i \cdot z_j \cdot a_k \cdot c}{BW_{ijk} \cdot \frac{Z_j^P}{V^P} \cdot \frac{Q_i \cdot A_k}{V} \cdot c} \right) \right] \to Min! \tag{32}$$

The (destination-) potentials Z_j^P will be justified involved in the calculation. By this step a consistent start solution arises containing the same terms in numerator and denominator except the increase- or correction factors. In turn these are the result of the compliance with the boundary sum criteria.

7 DETERMINATION OF MINIMAL AND MAXIMAL BOUNDARY SUM CRITERIA AND THE POTENTIALS

The legitimate question about the determination of minimal and maximal boundary sum criteria arises in this place if these are not known, for example by structure size specifications. In the simplest case the minimal and maximal boundary sum criteria could be chosen freely. An approach is also possible, which Lohse et al. (cp. [6] S. 16 ff.) use in their trip generation model. Shown at a simplified example, at first the general (destination-) traffic volume will be calculated as follows

$$Z_j = ER_j \cdot SG_j \cdot v_j \tag{33}$$

The overload factor *ue* used for the determination of the elastic (only maximum) boundary sum criteria can also be used for the determination of the minimal and maximal boundary sum criteria.

$$\begin{aligned} Z_j^{min} &= ER_j \cdot SG_j \cdot v_j \cdot ue_j^{min} \\ Z_j^{max} &= ER_j \cdot SG_j \cdot v_j \cdot ue_j^{max} \end{aligned} \tag{34}$$

ER Generation rates or attraction rates
SG Structure size
v Internal travel quota

At this a general problem of the transport planning gets visible, that none or only insufficient data are existing for the attraction side.

The potentials still have to be determined if these are not given, though. In turn several possibilities are conceivable. Besides the free determination of the potentials Z_j^P, these can also be decided on $Z_j^P = Z_j^{max}$. Furthermore a calculation of the potentials (e.g. mean average values from upper and low limits) is possible under circumstances, too.

$$Z_j^P = \frac{Z_j^{min} + Z_j^{max}}{\sum_j Z_j^{min} + \sum_j Z_j^{max}} \cdot V \tag{35}$$

8 FURTHER CONSIDERATIONS

The differences of the boundary sum criteria defined as hard, elastic and open up till now as well as the possibility of the mixture (cp. Figure 1) considerably complicate the renewed definition of the boundary sum criteria. The attempt of a definition shall not be made either, though. It is better to consider the real specifications of the minimal and maximal boundary sum criteria.

As an expansion of the introduced model a reaction to the utilisation of destinations was also formulated. The overload of a destination is behavior specifically and consequently parameterisable. If at a high demand only one single destination exists, for example, this is made full use up to 100% of its capacity, although it is more unattractive than at a moderate utilisation. If, in contrast, several alternative attractive destinations exists and they are not fully occupied, they are already chosen at increasing utilisation of a destination (for example at 80 -90 % utilisation). All alternative destination can also be fully occupied till 100%, however, depending on necessity.

The combination of the introduced model with other models is also possible. The faq_{ik}- and faz_{jk}-factors described by Schiller in [8], which ones act for the restriction of single modes in traffic analysis zones, (e. g.: modeling of the resting traffic), be able to be introduced to the model theory of the minimal and maximal boundary sum criteria as follows:

$$
\begin{aligned}
&v_{ijk} = BW_{ijk} \cdot fq_i \cdot fz_j \cdot fa_k \cdot faq_{ik} \cdot faz_{jk} \\
&\sum_j \sum_k v_{ijk} = Q_i \\
&\sum_j v_{ijk} \leq Q_{ik}^{max} \\
&\sum_i \sum_k v_{ijk} \geq Z_j^{min} \\
&\sum_i \sum_k v_{ijk} \leq Z_j^{max} \\
&\sum_i v_{ijk} \leq Z_{jk}^{max} \\
&\sum_i \sum_j v_{ijk} = A_k
\end{aligned}
\tag{36}
$$

A universal and consistent model for the simultaneous destination choice and mode choice of individuals results with that. With it moving and resting traffic can be calculated on the same homogeneous model theoretical basis. All boundary sum criteria can be free chosen. It is not relevant whether they related to the traffic analysis zone or to the traffic analysis zone in connection with a mode.

In conclusion it can be noticed that with this explanatory model all to date known boundary sum criteria can be calculated. The solution algorithm iterates speedily and stably. The explanations shown here and restricted to 3 dimensions (*i, j, k*) can as well be extended to n-linear systems of equations. The advantages of the minimal and maximal boundary sum criteria can only be assessed to date and must prove themselves in the reality. First attempts show very good results, though.

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