17th International Conference on the Application of Computer
Science and Mathematics in Architecture and Civil Engineering
K. Gürlebeck and C. Könke (eds.)
Weimar, Germany, 12–14 July 2006

SOLVABILITY EXPLORATION OF SEGMENTATION PROBLEM WITH LINEAR CONVOLUTION ALGORITHMS

V.A. Perepelitsa, F.B. Tebueva, and T.M. Shenkao

Karachai-Circassian State Technological Academy
Russia, 369000, Cherkessk, Stavropolskaya str., 36
E-mail: timshenkao@yandex.ru

Keywords: market segmentation, bipartite graph cover by stars, interval problem, vector problem, Pareto optima, unsolvability with linear convolution algorithm.

Abstract. *The paper is dedicated to decidability exploration of market segmentation problem with the help of linear convolution algorithms. Mathematical formulation of this problem represents interval task of bipartite graph cover by stars. Vertices of the first partition correspond to types of commodities, vertices of the second – to customers groups. Appropriate method is offered for interval problem reduction to two-criterion task that has one implemented linear convolution algorithm. Unsolvability of multicriterion (and, consequently, interval) market segmentation problem with the help of linear convolution algorithm is proved.*

1 GRAPH-THEORETICAL MODEL OF MARKET SEGMENTATION

Market segmentation concept is the key point in marketing theory. Market segmentation consists of division of markets into precise groups of customers. These customers are characterized by similar reaction at suggested product or set of marketing incentives [12].

In this paper graph-theoretical model is proposed as an adequate mathematical model for market segmentation problem. Its mathematical description is formulated under the assumption that a base for market segmentation is defined a priori. It means that the set of would-be users (individual customers, organizations) is given and segmentation factors (criteria) are chosen. Besides, a nomenclature of goods of the same type that are presented at the market is determined. In process of market segmentation modeling, mathematical formulation is stated as a multicriterion problem in a bipartite graph $G = (V_1, V_2, E)$ [7] with partitions' orders $|V_1| = m$ and $|V_2| = l$, $m \leqslant l$ [2].

Vertices $v_i \in V_1$ correspond one-to-one to suggested types of product; these types are enumerated with the index $i = \overline{1, m}$. Vertices $v_j \in V_2$ correspond one-to-one to customers groups; these groups are enumerated with the index $j = \overline{1, l}$.

By n_j denote the predicted quantity of commodity pieces purchased by representatives of the j^{th} group.

The edge $e = (v_i, v_j)$ belongs to the set E if and only if the i^{th} type of goods can be acceptable for customers from the j^{th} group ($1 \leqslant j \leqslant l$). Each edge $e \in E$ in the graph $G = (V_1, V_2, E)$ is weighted with numbers $w_\upsilon(e)$, $\upsilon = \overline{1, N}$. Weights $w_\upsilon(e)$ reflect expertly defined degree of consumer usefulness of the i^{th} type of product for customers from the j^{th} group, $0 \leq w_\upsilon(e) \leq 1$, $\upsilon = \overline{1, N}$, $e \in E$. Here index υ enumerates the criteria of consumer quality of product: longevity, reliability, convenience in exploitation, etc.

By k_i denote minimally admissible number of commodity's copies of the i^{th} type given a priori. If the production volume of the i^{th} type commodity is larger than k_i, then production of these goods comes out to be profitable $i = \overline{1, m}$.

A feasible solution of segmentation problem in bipartite graph $G = (V_1, V_2, E)$ is the subgraph $x = (V_1^x, V_2, E_x)$, $V_1^x \subseteq V_1$, $E_x \subseteq E$ of $G = (V_1, V_2, E)$. Every component of the subgraph is either an edge $e \in E$ or a star on $(h + 1)$ vertices $h \in \{2, 3, \ldots, l\}$ [7]. This star has center in some vertex $v_i \in V_1$ and its edges generate the set E_x^i, $i \in \{1, 2, \ldots, m\}$.

Dangling vertices of some concrete star E_x^i generate subset $V_2^x(v_i) \subseteq V_2$ that satisfies inequality

$$\sum_{v_j \in V_2^i} n_j \geq k_i, \quad v_i \in V_1^x, \tag{1}$$

where center $v_i \in V_1^x$, $i = 1, 2, \ldots, m$ and union $\bigcup\limits_{v_i \in V_1^x} V_2^x(v_i) = V_2$.

Remark 1. If an edge $e = (v_i, v_j)$ belongs to E_x, then, taking into consideration condition (1), it can be viewed as a star on 2 vertices, with center v_i and dangling vertex v_j. We do not consider condition (1) for vertices $v_i \in (V_1 \backslash V_1^x)$.

An illustrative example of bipartite graph G and feasible solution $x \in X(G)$ is presented for given parameters in figure 1a) and 1b). This parameters correspond to condition (1).

We denote by $X = X(G) = \{x\}$ a set of all feasible solutions (SFS) in graph $G = (V_1, V_2, E)$. Vector objective function (VOF) is defined on SFS X

$$F(x) = (F_1(x), F_2(x), \ldots, F_{N+1}(x)), \tag{2}$$

and consists of N criteria of the weight form $MAXSUM$

$$F_\upsilon(x) = \sum_{e \in E_x} w_\upsilon(x) \to \max, \quad \upsilon = \overline{1, N} \tag{3}$$

and one criterion of combinatorial kind

$$F_{N+1}(x) = |V_1^x| \to \max, \quad \upsilon = \overline{1, N}; \tag{4}$$

this criterion reflects variety of goods, i.e., the number of types (of commodity) that is profitable to produce.

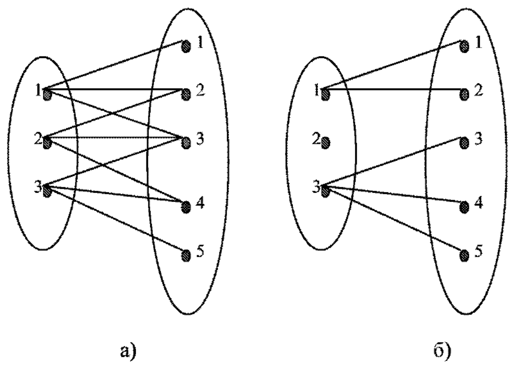

Figure 1: a) Bipartite graph $G = (V_1, V_2, E)$, $|V_1| = m = 3$, $|V_2| = l = 5$ and b) example of feasible solution $x = (V_1^x, V_2, E)$, $V_1^x \subset V_1$, $E_x \subset E$ for given parameters $k_1 = 2$, $k_2 = k_3 = 3$, $n_j = 1$, $j = \overline{1, 5}$.

VOF (2)-(3) defines on SFS X Pareto set (PS) $\widetilde{X}$, which consists of all Pareto optima (PO) $\widetilde{x} \in \widetilde{X}$ [2].

Every pair of PO $\widetilde{x}_1, \widetilde{x}_2 \in \widetilde{X}$ is considered to be equivalent if an equality takes place: $F(\widetilde{x}_1) = F(\widetilde{x}_2)$. So, in this work, we deal with an algorithmic problem of finding so called complete set of alternatives (CSA) [2].

The subset $X^0 \subseteq \widetilde{X}$ is called CSA if its cardinality $|X^0|$ is minimal when equation is held: $F(X^0) = F\left(\widetilde{X}\right)$, where $F(X^*) = \{F(x) : x \in X^*\}\ \forall X^* \subseteq X$. Here and later we mean that PS $\widetilde{X}$ and CSA X^0 are determined for given graph G, i.e., $\widetilde{X} = \widetilde{X}(G)$, $X^0 = X^0(G)$.

2 INTERVAL SEGMENTATION PROBLEM STATEMENT

The values of the weights $w_\upsilon(e)$, $\upsilon = \overline{1, N}$ prescribed by experts [6] are of an approximate kind in real circumstances. So, it is possible to use the apparatus of interval calculus [1] to reflect the uncertainty of such kind in mathematical model.

Let minimally possible $w_1(e)$ and maximally possible $w_2(e)$ values of segmentation factor $w(e)$ be known. Then, the weight of the edge $e \in E$ can be represented as interval $w(e) = [w_1(e), w_2(e)]$, $w_1(e) \leqslant w_2(e)$.

We give the definition of interval segmentation problem as a problem of a bipartite graph $G = (V_1, V_2, E)$ cover by stars on h vertices, $h = \{2, 3, \ldots, l\}$, for one-criterion ($N = 1$) case.

Bipartite graph $G = (V_1, V_2, E)$ is given, its partitions' orders are $|V_1| = m$ and $|V_2| = l$. Each edge $e \in E$ of graph G is weighted with interval $w(e) = [w_1(e), w_2(e)]$, $w_1(e) \leqslant w_2(e)$.

The feasible solution of segmentation problem formulated in bipartite graph $G = (V_1, V_2, E)$ is some part $x = (V_1^x, V_2, E_x)$, $V_1^x \subseteq V_1$, $E_x \subseteq E$ of graph $G = (V_1, V_2, E)$. Every component of this part is either an edge incident to vertices of the first V_1 and the second V_2 partitions or a star on $(h+1)$ vertices, $h = \{2, 3, \ldots, l\}$, with center in vertex $v_i \in V_1$ and edges E_x^i, $i \in \{1, 2, \ldots, m\}$.

At the same time, dangling vertices of concrete star E_x^i generate subset $V_2^x(v_i) \subseteq V_2$ and this subset satisfies inequality (1).

Vector objective function is defined on SFS X so that interval weights are assigned

$$F(x) = \{W(x), |V_1^x|\}, \tag{5}$$

Vector objective function consists of weight criterion of the form $MAXSUM$

$$W(x) = \sum_{e \in E_x} w(e) \to \max, \tag{6}$$

where $w(e) = [w^1(e), w^2(e)]$, $w^1(e) \leqslant w^2(e)$, $W^i(x) = \sum\limits_{e \in E_x} w^i(e)$, $i = 1, 2$, and one criterion of the combinatorial form (similar to (4))

$$|V_1^x| \to \max; \tag{7}$$

this criterion represents variety of goods.

3 REDUCTION OF GRAPH COVER BY STARS INTERVAL PROBLEM TO VECTOR PROBLEM

To solve optimization problem with data in interval representation, we consider reduction of this problem to derived two-criterion problem stated below. We formulate statements of both problems in bipartite graph $G = (V_1, V_2, E)$.

Bipartite graph $G = (V_1, V_2, E)$ is given and each edge of it is weighted with some interval $w(e) = [w^1(e), w^2(e)]$, $w^1(e) \leqslant w^2(e)$. On SFS X, which is defined by this graph, interval objective function (IOF) of the weight form $MAXSUM$ is specified:

$$W(x) = \sum_{e \in E_x} w(e) \to \max, \tag{8}$$

where $w\left(e\right)=\left[w^{1}\left(e\right),w^{2}\left(e\right)\right]$, $w^{1}\left(e\right)\leqslant w^{2}\left(e\right)$, $W^{i}(x)=\sum\limits_{e\in E_{x}}w^{i}(e)$, $i=1,2$.

The summation in this IOF is fulfilled subject to properties of interval addition operations [1, 10].

The decision of interval problem is an element $x^{0}\in X$; IOF (8) attains necessary extreme value at this element. However, among values $W\left(x\right)$, $x\in X$ of IOF (8), the mentioned extreme value can be missing in general case if SFS X contains noncomparable elements. To choose the most appropriate solution from the set of noncomparable alternatives, it is needed to introduce the definitions of relations of preference, equivalence and incomparability [10].

Of two decisions x_{1} and x_{2}, $x_{1},x_{2}\in X$, x_{1} is more preferable than x_{2} ($x_{1}\prec x_{2}$) if $W^{i}\left(x_{1}\right)\leqslant W^{i}\left(x_{2}\right)$, $i=1,\,2$ and at least one of these inequalities is absolute.

Solutions x_{1} and x_{2} are incomparable ($x_{1}\approx x_{2}$) if strict interval nesting takes place: $W^{i}\left(x_{1}\right)\subset W^{i}\left(x_{2}\right)$ or $W^{i}\left(x_{2}\right)\subset W^{i}\left(x_{1}\right)$.

Solutions x_{1} and x_{2} are equivalent ($x_{1}\equiv x_{2}$) such that strict interval coincidence takes place: $W^{i}\left(x_{1}\right)=W^{i}\left(x_{2}\right)$.

The relations of preference and incomparability generate on SFS X Pareto set (PS) $\widetilde{X}\subseteq X$ consisted of Pareto optima (PO). Let us introduce the concepts of PO and CSA for considered interval problem.

For the problem with IOF (8), solution $x\in X$ is called PO if it does not exist such an element $x^{*}\in X$ that $x^{*}\prec\widetilde{x}$.

The CSA is the subset of minimal cardinality containing one representative for each value $W(x)$, $x\in\widetilde{X}$, where $W(x)$ is the value of IOF (8).

Now we can formulate two-criterion task, which is derived from earlier presented interval problem with IOF (8).

SFS X is specified in bipartite graph $G=(V_{1},V_{2},E)$ with edges weighted by pair of weights $w_{1}\left(e\right)$ and $w_{2}\left(e\right)$, $w_{1}\left(e\right)\leqslant w_{2}\left(e\right)$. Vector objective function (VOF) is defined on SFS X:

$$F\left(x\right)=\left(F_{1}\left(x\right),F_{2}\left(x\right)\right), \tag{9}$$

and consists of 2 criteria of the weight form $MAXSUM$

$$F_{\upsilon}(x)=\sum_{e\in E_{x}}w_{\upsilon}(e)\rightarrow\max,\quad\upsilon=1,2. \tag{10}$$

VOF (9)-(10) determines on SFS X PS $\widetilde{X}$ and CSA X^{0} [3], where CSA X^{0} is taken as a sought solution of formulated two-criterion problem.

In paper [8], statement is substantiated that every interval problem in graphs with IOF (8) is equivalent to corresponding derived two-criterion problem with VOF (9)-(10) under the assumption

$$w^{1}\left(e\right)=w_{1}\left(e\right),\quad w^{2}\left(e\right)=w_{2}\left(e\right). \tag{11}$$

Thus, two formulated earlier problems are equivalent, i.e., VOF (9)-(10) and IOF (8) determine on SFS of these tasks coincident PO and CSA. Stated equivalency gives an opportunity to hold single exploration of algorithmic issues of segmentation problem for vector (2)-(3) and interval (5)-(6) representations of figure of merit.

4 LINEAR CONVOLUTION ALGORITHM DESCRIPTION

Linear convolution algorithms [2, 9] are the most popular among the methods of determination Pareto optimal solutions for vector problems, i.e., elements $x \in \widetilde{X}$. These algorithms are based on the fact that element $x \in X$ maximizing (minimizing) convolution for positively defined VOF (9)-(10)

$$F^{\lambda}\left(x\right)=\sum_{\upsilon=1}^{N}\lambda_{\upsilon}\cdot F_{\upsilon}\left(x\right), \tag{12}$$

is Pareto optimal. Here vector is $\lambda \in \Lambda_N$, where $\Lambda_N = \left\{\lambda=(\lambda_1,\lambda_2,\ldots,\lambda_N) : \sum\limits_{\upsilon=1}^{N}\lambda_{\upsilon}=1,\right.$ $\left.\lambda_{\upsilon}>0,\ \upsilon=1,2,\ldots,N\right\}$.

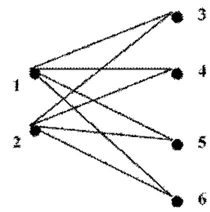

Figure 2: Graph $G^{*}=(V_1^{*},V_2^{*},E^{*})$.

Let us examine some individual task with N maximized criteria (3) that are determined on SFS $X=\{x\}$. We denote the sought set of alternatives (SA) of this problem by X^*, $X^*\subseteq X$. If there is a vector $\lambda^*\in\Lambda_N$ for every element $x^*\in X^*$ and this vector satisfies equation $F^{\lambda^*}(x^*)=\max\limits_{x\in X}F^{\lambda^*}(x)$, then one says that the problem of finding SA X^* is solvable with linear convolution algorithm. If solvability defined in such a way is typical for all individual problems of the examined mass problem [4], then for each of them one can find the sought SA with linear convolution algorithm. This problem is unsolvable with linear convolution algorithm if there is, for the examined task, individual problem with SA X^* containing such element $x^*\in X^*$ that at this element convolution $F^{\lambda}(x)$, $\forall\lambda\in\Lambda_N$ does not attain required extremum, i.e., for all $\lambda\in\Lambda_N$ inequality $F^{\lambda}(x^*)<\max\limits_{x\in X}F^{\lambda^*}(x)$ is held.

A range of papers (see references in [5]) are dedicated to the problem of solvability of certain multicriterion problems with linear convolution algorithms. However, investigations of solvability of the examined interval and corresponding vector problem (interval problem is reduced to it) are absent. We also note that the assertions about unsolvability of graph cover by stars problem with linear convolution algorithm obtained in paper [11] do not affect the considered formulations of market segmentation problem.

5 SUBSTANTIATION OF UNSOLVABILITY OF SEGMENTATION PROBLEM WITH LINEAR CONVOLUTION ALGORITHM

Let us denote by Z_1 concrete individual interval problem of full bipartite graph $G^* = (V_1^*, V_2^*, E^*)$ cover that has partitions' orders $|V_1^*| = m = 2$ and $|V_2^*| = l = 4$ (figure 2) with IOF (8).

The set of vertices of the first partition is $V_1^* = \{1, 2\}$, the set of vertices of the second partition is $V_2^* = \{3, 4, 5, 6\}$. The set $E^* = \{e_i\}$, $i = \overline{1, 8}$ consists of the edges: $e_1 = (1, 3)$, $e_2 = (1, 4)$, $e_3 = (1, 5)$, $e_4 = (1, 6)$, $e_5 = (2, 3)$, $e_6 = (2, 4)$, $e_7 = (2, 5)$, $e_8 = (2, 6)$.

Every edge $e \in E^*$ of the graph G^* is weighted by interval weights $w(e) = (w^1(e), w^2(e)) \subset [0; 1]$:

$$\begin{aligned} &w(e_1) = (0,02;\ 0,09), && w(e_2) = (0,06;\ 0,1), \\ &w(e_3) = (0,04;\ 0,09), && w(e_4) = (0,05;\ 0,25), \\ &w(e_5) = (0,1;\ 0,12), && w(e_6) = (0,07;\ 0,08), \\ &w(e_7) = (0,08;\ 0,16), && w(e_8) = (0,15;\ 0,2). \end{aligned} \tag{13}$$

Let us reduce this interval problem to two-criterion problem with VOF of the form (9)-(10). We get two-weighted graph $G = (V_1, V_2, E)$ that is isomorphic to the graph $G^* = (V_1^*, V_2^*, E)$, every edge of which is weighted by corresponding weights according to (11), (13):

$$\begin{aligned} &w_1(e_1) = 0,02; && w_2(e_1) = 0,09; && w_1(e_2) = 0,06; && w_2(e_2) = 0,1; \\ &w_1(e_3) = 0,04; && w_2(e_3) = 0,09; && w_1(e_4) = 0,05; && w_2(e_4) = 0,25; \\ &w_1(e_5) = 0,1; && w_2(e_5) = 0,12; && w_1(e_6) = 0,07; && w_2(e_6) = 0,08; \\ &w_1(e_7) = 0,08; && w_2(e_7) = 0,16; && w_1(e_8) = 0,15; && w_2(e_8) = 0,2. \end{aligned}$$

The SFS of individual problem is the set $X = \{x_r\}$, $x_r = (V_1^{x_r}, V_2, E_{x_r})$. The cardinality of SFS is $|X| = 16$, i.e., $r = 1, 2, \ldots, 16$.

Let us display all feasible decisions x_r that are defined by corresponding sets of edges E_{x_r}:

$$\begin{aligned} &E_{x_1} = \{e_1, e_6, e_7, e_8\}, && E_{x_2} = \{e_2, e_5, e_7, e_8\}, && E_{x_3} = \{e_3, e_5, e_6, e_8\}, \\ &E_{x_4} = \{e_4, e_5, e_6, e_7\}, && E_{x_5} = \{e_1, e_4, e_6, e_7\}, && E_{x_6} = \{e_2, e_3, e_5, e_8\}, \\ &E_{x_7} = \{e_1, e_2, e_3, e_8\}, && E_{x_8} = \{e_1, e_2, e_4, e_7\}, && E_{x_9} = \{e_1, e_3, e_4, e_6\}, \\ &E_{x_{10}} = \{e_2, e_3, e_4, e_5\}, && E_{x_{11}} = \{e_1, e_3, e_6, e_8\}, && E_{x_{12}} = \{e_3, e_4, e_5, e_6\}, \\ &E_{x_{13}} = \{e_1, e_2, e_7, e_8\}, && E_{x_{14}} = \{e_2, e_4, e_5, e_7\}, && E_{x_{15}} = \{e_1, e_2, e_3, e_4\}, \\ &E_{x_{16}} = \{e_5, e_6, e_7, e_8\}. \end{aligned}$$

We compute the value of criteria for listed decisions $F_\upsilon(x_r) = \sum\limits_{e \in E_{x_r}} w_r(e)$, $\upsilon = 1, 2$, VOF (9)-(10) (see Table 1).

According to Table 1, one can conclude about vector incomparability of solutions x_2, x_3, x_4, x_6, x_{14}. These five decisions generate PS with which CSA coincide: $\widetilde{X} = X^0 = \{x_2, x_3, x_4, x_6, x_{14}\}$.

Table 1.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$F_1(x_r)$	0,24	0,31	0,36	0,30	0,22	0,35	0,27	0,21
$F_2(x_r)$	0,53	0,58	0,49	0,61	0,58	0,51	0,48	0,60

	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}
$F_1(x_r)$	0,18	0,25	0,28	0,26	0,23	0,29	0,17	0,32
$F_2(x_r)$	0,51	0,56	0,46	0,54	0,55	0,63	0,53	0,56

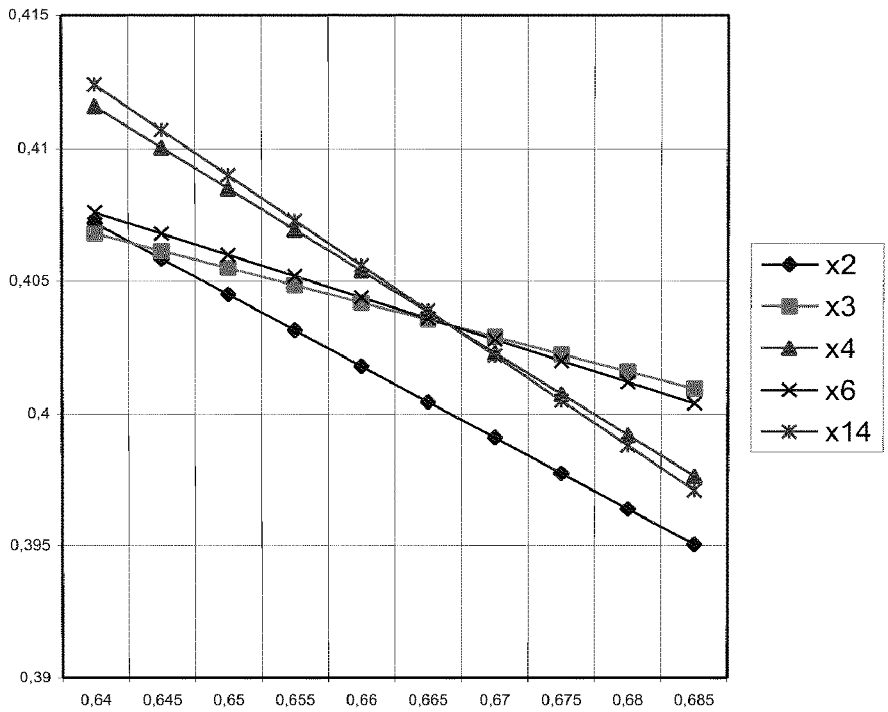

Figure 3: Graphic presentation of convolutions $F(x_r, \lambda_1)$, $r = 2,\ 3,\ 4,\ 6,\ 14$.

Then, we form convolutions (10) according to (12); taking into consideration these convolutions and $\lambda_1 + \lambda_2 = 1$, $\lambda_2 = 1 - \lambda_1$, we can state :

$$\begin{aligned}
F^{\lambda}(x_2) &= F(x_2, \lambda_1) = 0,31\lambda_1 + 0,58\lambda_2 = 0,58 - 0,27\lambda_1 \\
F^{\lambda}(x_3) &= F(x_3, \lambda_1) = 0,36\lambda_1 + 0,49\lambda_2 = 0,49 - 0,13\lambda_1 \\
F^{\lambda}(x_4) &= F(x_4, \lambda_1) = 0,30\lambda_1 + 0,61\lambda_2 = 0,61 - 0,31\lambda_1 \\
F^{\lambda}(x_6) &= F(x_6, \lambda_1) = 0,35\lambda_1 + 0,51\lambda_2 = 0,51 - 0,16\lambda_1 \\
F^{\lambda}(x_{14}) &= F(x_{14}, \lambda_1) = 0,29\lambda_1 + 0,63\lambda_2 = 0,63 - 0,34\lambda_1
\end{aligned} \tag{14}$$

There is graphical representation of these convolutions $F^{\lambda}(x) = F(x, \lambda_1)$ as functions of λ_1 in figure 3.

From this graphical representation of convolutions $F(x_2, \lambda_1)$, $F(x_3, \lambda_1)$, $F(x_4, \lambda_1)$, $F(x_6, \lambda_1)$, $F(x_{14}, \lambda_1)$, it is clear that convolutions graphs $F(x_{14}, \lambda_1)$, $F(x_3, \lambda_1)$ make upper bound. All the rest graphs are situated strictly below this bound. Here we come to conclusion that maximal

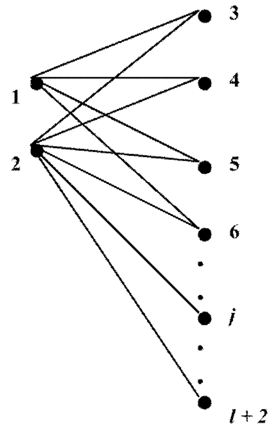

Figure 4: Graph $G^*_{2,l} = \left(V_1^*, V_2^{*l}, E^{*2,l}\right)$.

value of convolutions $F^\lambda(x_r)$, $r = 2, 4, 6$ does not attain this bound under no values $\lambda \in \Lambda$. Now we formulate 2 lemmas based on conducted constructive proof of unsolvability of individual bipartite graph $G = (V_1, V_2, E)$ cover by stars on h vertices problem with linear convolution algorithm for the case of IOF (8) and VOF (9)-(10).

Lemma 1. *Interval problem of cover by stars on h vertices, $h \in \{2, 3, 4\}$, of the bipartite graph $G^* = (V_1^*, V_2^*, E^*)$, $|V_1^*| = 2$, $|V_2^*| = 4$ weighted according to (13) with IOF (8) is unsolvable with linear convolution algorithm.*

Lemma 2. *Derived (from problem stated in lemma 1) vector problem of bipartite graph $G = (V_1, V_2, E)$, $|V_1| = 2$, $|V_2| = 4$ cover by stars on h vertices, $h \in \{2, 3, 4\}$, with VOF (9)-(10) is unsolvable with linear convolution algorithm.*

We prove some lemma for obtained results generalization.

Lemma 3. *Interval problem of cover by stars on h vertices, $h \in \{2, 3, \ldots, l\}$ of bipartite graph $G^*_{2,l} = \left(V_1^*, V_2^{*l}, E^{*2,l}\right)$, $|V_1^*| = 2$, $\left|V_2^{*l}\right| = l$ weighted according to (13) with IOF (8) is unsolvable with linear convolution algorithm.*

Proof. We extend lemma's 1 proof for the case $l > 4$. For that, we transform the graph $G^* = \left(V_1^*, V_2^{*l}, E^*\right)$, $|V_1^*| = 2$, $|V_2^*| = 4$ to graph $G^*_{2,l} = \left(V_1^*, V_2^{*l}, E^{*2,l}\right)$ with partitions' orders $|V_1^*| = 2$, $\left|V_2^{*l}\right| = l$ by entering $(l-4)$ additional vertices v_j in second partition's V_2^* set of vertices. The set of edges is:

$$E^{*2,l} = E^* \cup \Delta E^*, \text{ where } \Delta E^* = \{e = (2, j)\}, \quad j = 7, 8, \ldots, l+2, \tag{15}$$

i.e., cardinality $|\Delta E^*| = l - 4$ (figure 4). Denote derived problem by symbol Z_2 and its SFS by symbol $X_{2,l} = \left\{x_r^{2,l}\right\}$.

Remark 2. It is easy to notice a bijection between elements of SFS $X = \{x_r\}$, $r = 1, 2, \ldots, 16$ of problem Z_1 and SFS $X_{2,l} = \left\{x_r^{2,l}\right\}$, $r = 1, 2, \ldots, 16$ of problem Z_2.

In fact, if the feasible solution $x_r = (V_1, V_2, E_{x_r}) \in X$ consists of two stars $E^1_{x_r}$ and $E^2_{x_r}$ with centers respectively in vertices 1 and 2 of the first partition V_1^* of graph G^*, then corresponding admissible decision $x_r^{2,l} = \left(V_1^*, V_2^{*l}, E_r^{2,l}\right)$ also consists of two stars $E_r^{2,l,1}$, $E_r^{2,l,2}$ with centers respectively in vertices 1 and 2 of the first partition of the graph $G^*_{2,l}$. In other words, the first stars coincide in these feasible solutions and the second star of decision $x_r^{2,l}$ is obtained by the addition of the set of edges ΔE^* to the second star of solution $x_r \in X$.

Interval unit weight $w(e)$ is assigned to every edge $e \in \Delta E^* = \left(E^*_{2,l} \backslash E^*\right)$ that is incident to any of the added vertices $v_j \in V_2^*$, i.e., the length of interval $w(e) = (w^1(e), w^2(e))$ is zero: $w^1(e) = w^2(e) = 1$.

Let us use the same reasoning that was performed when we proved lemmas 1 and 2. We compute the values of criteria $F_v\left(x_r^{2,l}\right) = \sum\limits_{e \in E^*_{2,l}} w_r(e)$, $v = 1, 2$. The found values of criteria $F_v\left(x_r^{2,l}\right)$, $v = 1, 2$, that compose VOF (9)-(10), differ from corresponding values in Table 1 in constant $c = l - 4$. So, the increase of the second partition's cardinality l does not change PS and CSA for the task Z_2 in comparison with the task Z_1. Making linear convolutions, we get formulas that differ from similar ones (14) in constant c. Hence, graphical representation of these convolutions $F^\lambda(x) = F(x, \lambda_1)$ as functions against λ_1 differ from figure 3 by bias of all graphs upwards in constant c; this distinction does not affect correctness of conclusions of lemma 1. $\square$

Lemma 4. *Interval problem of cover by stars on h vertices, $h \in \{2, 3, \ldots, l\}$, of bipartite graph weighted according to (13) $G^*_{m,l} = \left(V_1^{*m}, V_2^{*l}, E^{*m,l}\right)$, $|V_1^{*m}| = m$, $\left|V_2^{*l}\right| = l$ with IOF (8) is unsolvable with linear convolution algorithm.*

Proof. We inspect the graph $G^*_{m,l} = \left(V_1^{*m}, V_2^{*l}, E^{*m,l}\right)$ on $(m + l)$ vertices, $|V_1^{*m}| = m$, $\left|V_2^{*l}\right| = l$, which was obtained on a base of bipartite graph $G^*_{2,l} = \left(V_1^*, V_2^{*l}, E^{*2,l}\right)$, $|V_1^*| = 2$, $\left|V_2^{*l}\right| = l$ by completion of its first partition V_1^* with additional vertices v_i, $i = \overline{l+3, l+m}$. Then, the set $E^{*2,l}$ of edges of the graph $G^*_{2,l}$ is enlarged with $(m - 2)$ edges of the kind $e = (i, 3)$, $i = \overline{l+3, l+m}$. We assign zero intervals $w(e) = \underline{(w^1(e), w^2(e)) =} (0, 0)$ to these edges. The set of these edges we denote by $E^0 = \{e = (i, 3)\}$, $i = \overline{l+3, l+m}$.

Let denote interval problem of the examined graph $G^*_{m,l}$ cover by stars by Z_3. The SFS $X_{m,l}$ of this task can be divided into 2 subsets

$$X_{m,l} = X^1_{m,l} \cup X^2_{m,l}, \tag{16}$$

where $X^1_{m,l}$ consists only of those feasible solutions that have two components. These components are the stars with centers in vertices 1 and 2 of the first partition.

It follows from this definition that one-to-one correspondence exists between feasible solutions $x_r^{m,l} \in X^1_{m,l}$ and feasible solutions $x_r^{2,l} \in X_{2,l}$ of the task Z_2. At the same time, decision $x_r^{m,l}$ consists of 2 stars that constitute solution $x_2^{m,l}$ and of $(m - 2)$ isolated vertices, $i = \overline{l+3, l+m}$. From edge weight definition $e \in E^{*m,l}$ and separation (16), it is clear that every decision from subset $X^2_{m,l}$ is other than Pareto optimal.

Thus, from given statements, it turns out that the problem Z_3 inherits PS and CSA of the task Z_2 in some strictly determined sense. Forming linear convolutions on these PS and CSA, we get expressions that completely coincide with linear convolutions of the kind (14) for the task Z_2. Using graphical representation of these convolutions, we get the proof of lemma 4 analogously to lemma 3. $\square$

Substituting in proofs of lemma 3 and lemma 4 $w^1(e) = w_1(e)$, $w^2(e) = w_2(e)$ and formulating problems Z_4, Z_5 analogous correspondingly to problems Z_2, Z_3, we come to conclusion that the next lemma 5 and lemma 6 are correct.

Lemma 5. *Derived (from problem presented in lemma 1) vector problem of bipartite graph $G = (V_1, V_2, E)$, $|V_1| = 2$, $|V_2| = l$ cover by stars on h vertices, $h \in \{2, 3, \ldots, l\}$, with VOF (9)-(10) is undecidable with the help of linear convolution algorithm.*

Lemma 6. *Derived (from problem presented in lemma 1) vector problem of bipartite graph $G = (V_1, V_2, E)$, $|V_1| = m$, $|V_2| = l$ cover by stars on h vertices, $h \in \{2, 3, \ldots, l\}$, with VOF (9)-(10) is undecidable with the help of linear convolution algorithm.*

Utilizing received results, let get back to segmentation problem in interval and multicriterion formulation.

Theorem 1. *Interval segmentation problem with regard to inequality (1) and IOF (5)-(7) is unsolvable with linear convolution algorithm.*

Proof. It is necessary to extend the result obtained in lemma 4 with regard to criterion of combinatorial kind (7) that is about maximal number of connected components in the graph $G^*_{m,l} = \left(V_1^{*m}, V_2^{*l}, E^{*m,l}\right)$ with partitions' cardinalities $|V_1^{*m}| = m$, $\left|V_2^{*l}\right| = l$.

Let us consider the graph $G^*_{m,l} = \left(V_1^{*m}, V_2^{*l}, E^{*m,l}\right)$ on $(m + l)$ vertices with partitions' cardinalities $|V_1^{*m}| = m$, $\left|V_2^{*l}\right| = l$, which was received on a base of the bipartite graph $G^* = (V_1^*, V_2^*, E^*)$, $|V_1^*| = 2$, $|V_2^*| = 4$, that was discussed in the task Z_1, by completion of its first partition V_1^* with additional vertices v_i, $i = 7, 8, \ldots, m+4$ and entering $(l - 4)$ additional vertices v_j, $j = \overline{m + 5, m + l}$ into the set of vertices of the second partition V_2^*. Then, we enlarge the set E^* of edges of the graph G^* with the set of edges E^l so that one of the added vertices v_i of the first partition V_1^* becomes the center of the star on $(l - m)$ vertices with edges incident to the added vertices v_j, $j = \overline{m + 5, m + l}$ of the second partition V_2^*, and the remaining $(m - 3)$ vertices of the first and the second partitions are connected by edges. The cardinality of the set is $\left|E^l\right| = (l - 4)$. So, the set of edges of the graph $G^*_{m,l} = \left(V_1^{*m}, V_2^{*l}, E^{*m,l}\right)$ is the union of two sets $E^{*m,l} = E^1 \cup E^*$. We assign interval unit weight $w(e) = (w^1(e), w^2(e)) = (1, 1)$ to the edges e of the set E^1. Consequently, the received graph $G^*_{m,l} = \left(V_1^{*m}, V_2^{*l}, E^{*m,l}\right)$, $|V_1^{*m}| = m$, $\left|V_2^{*l}\right| = l$ consists of $((m - 2) + l)$ connected components: one star on $(l - m)$ vertices, graph $G^* = (V_1^*, V_2^*, E^*)$, $|V_1^*| = 2$, $|V_2^*| = 4$, and $(m - 3)$ edges $e \in E_l$. The formulated problem we denote by Z_6.

Let us consider SFS $X^{Z_6} = \left\{x_r^{Z_6}\right\}$ of the problem Z_6. According to the definition of SFS X of market segmentation problem, it can be concluded that all solutions $x_r^{Z_6}$ contain all $((m - 2) + 1)$ augmented connected components of the graph $G^*_{m,l}$ and one feasible solution x_r of the task Z_1. Then, holding reasoning analogous to ones that were made when solving problems Z_1 and Z_2, we find PS with which CSA coincides: $\widetilde{X} = X^0 = \{x_2,\, x_3,\, x_4, x_6,\, x_{11}\}$.

Now it is necessary to optimize this solution according to criterion (7). Maximal number of stars for the given graph $G^*_{m,l} = \left(V_1^{*m}, V_2^{*l}, E^{*m,l}\right)$, $|V_1^{*m}| = m$, $\left|V_2^{*l}\right| = l$ is equal to cardinality of the set of vertices of the first partition $|V_1^{*m}| = m$. All found solutions $x_r^{Z_6} \in X^{Z_6}$ fit this criterion. So, inferences from lemma 1 are correct with regard to criterion (7).

It is necessary to prove the obtained result with regard to inequality (1). Let us assume $k_i = n_j = c_1 = const$. Under this suggestion, the condition (1) would be fulfilled on all SFS $X^{Z_6} = \left\{x_r^{Z_6}\right\}$ of the task Z_6. So, the result obtained above remains in force. □

Lemma 7. *Vector problem of bipartite graph $G = (V_1, V_2, E)$, $|V_1| = m$, $|V_2| = l$ cover by stars on h vertices, $h = 2, 3, \ldots, l$, with VOF (2)-(4) is unsolvable with linear convolution algorithm.*

Proof. Basing on the results of lemma 6, we can infer that lemma 7 is proved for the case $N = 2$.

When $N > 2$ we admit for $i > 2$ the weights from G^*. After linear convolution algorithm usage, the same result as in the task Z_5 is obtained. □

Theorem 2. *Multicriterion segmentation problem under condition (1) and with VOF (2)-(4) is unsolvable with convolution algorithm.*

Proof. Lemma 7 proves unsolvability of segmentation problem as the problem of bipartite graph $G = (V_1, V_2, E)$, $|V_1| = m$, $|V_2| = l$ cover by stars on h vertices, $h = 2, 3, \ldots, l$, with VOF (2)-(4). In some cases the embedding of restriction (1) does not change neither PS nor CSA, and it was shown in theorem's 1. Therefore, theorem 2 is proved. □

6 CONCLUSION

In this paper explorations were conducted to find out the applications of convolution algorithms for market segmentation problem. According to the results of the research, one can make a decision that every method based on convolution algorithm cannot guarantee the obtaining of precise solution for market segmentation problem both in interval and multicriterion formulations. Pareto optima received with convolution algorithm in general case generate eigensubset of the sought CSA. In other words, convolution algorithm usage gives an opportunity to get the approximation of the sought set of solutions for segmentation problem in interval and multicriterion statements.

REFERENCES

[1] Alefeld, G. and Herzberger, J., *Introduction to Interval Computations*, New York, Academic Press, 1983.

[2] Emelichev, V.A. and Perepelitsa, V.A., Complexity of Vector Optimization Problems on Graphs, Optimization 22 (1991) 6, 903-918.

[3] Emelichev, V.A. and Perepelitsa, V.A., To Computational Complexity of Discrete Multicriterion Problems, *Izvestia of Academy of Sciences of USSR. Tekhn. Cybernetica* 1 (1988), pp. 85-87 (in Russian).

[4] Garey, M. and Johnson, D. S., *Computers and Intractability. A Guide to the Theory of NP-completeness*, San Francisco, W. H. Freeman and Co., 1979.

[5] Girlich, E., Perepelitsa, V.A., and Sergeeva, L.N., Research of solvability of vector optimization problems by means of the linear convolution algorithm, Preprint 7, Fakultät für Mathematik, Otto-von-Guericke-Universität, Magdeburg, 1994.

[6] Litvak, B.G., *Expert Information: Acquisition Methods and Analysis*, Moscow, Radio i Svyaz, 1984 (in Russian).

[7] Melnikov, O., Tyshkevich, R., Emelichev, V., and Sarvanov, V., *Lectures on Graph Theory*. B.I. Wissenschaftsverlag, Mannheim, Leipzig, Wien, Zurich, 1994.

[8] Perepelitsa, V.A. and Kozina, G.L., Interval Discrete Models and Multiobjectivity. Complexity Estimates, *Interval Computations* 1 (1993), pp. 51-59.

[9] Perepelitsa, V.A. and Sergienko, I.V., Investigation of a Class of Integer Multiobjective Problems, *J.Comp.Mathem. and Math.Phys.* 28 (3) (1988), pp. 400-419 (in Russian).

[10] Voshchinin, A.P. and Sotirov, G.R., *Optimization under Uncertainty*, Moscow, Sofia, MEI – Tekhnika, 1989 (in Russian).

[11] Zakhovalko, T.V. and Ryabenko, A.E., Decidability Exploration of Graph Cover by Stars Interval Problem with Linear Convolution Algorithm, *Proceedings of Applied Mathematics and Mathematical Modeling*, Dnepropetrvsk, RVV DNU, 2003, pp. 55-65 (in Russian).

[12] Zozulev, V.S., *Market Segmentation*, Moscow, Nauka, 1990 (in Russian).