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THE INFLUENCE OF CRACKS AND OVERESTIMATION ERRORS ON THE DEFLECTION OF THE REINFORCED CONCRETE BEAMS IN THE RIGID FINITE ELEMENT METHOD

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Abstract. *This article presents the Rigid Finite Element Method in the calculation of reinforced concrete beam deflection with cracks. Initially, this method was used in the shipbuilding industry. Later, it was adapted in the homogeneous calculations of the bar structures. In this method, rigid mass discs serve as an element model. In the flat layout, three generalized coordinates (two translational and one rotational) correspond to each disc. These discs are connected by elastic ties. The genuine idea is to take into account a discrete crack in the Rigid Finite Element Method. It consists in the suitable reduction of the rigidity in rotational ties located in the spots, where cracks occurred. The susceptibility of this tie results from the flexural deformability of the element and the occurrence of the crack. As part of the numerical analyses, the influence of cracks on the total deflection of beams was determined. Furthermore, the results of the calculations were compared to the results of the experiment. Overestimations of the calculated deflections against the measured deflections were found. The article specifies the size of the overestimation and describes its causes.*

1 INTRODUCTION

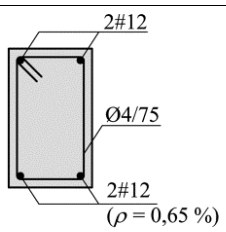
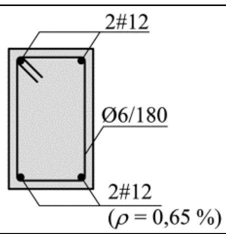
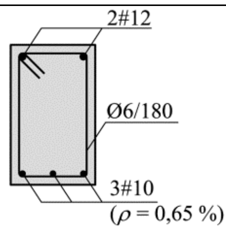
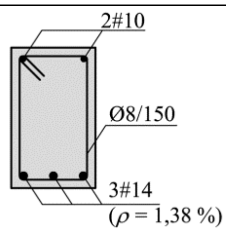
A phenomenon characteristic of reinforced concrete elements subjected to bending is the formation of cracks perpendicular to the axis of the tension zone. Cracks in concrete occur after the tension exceeds the tensile strength of concrete and they have fundamental influence on its operation. They cause, among others, increase in deflections [1]. In addition, in hyperstatic structures they are associated with redistribution of internal forces. In this paper, the rigid finite element method [2] was used in the computational analyses, which enables taking into account the cracks of the reinforced concrete element in a discreet way. A detailed description of the method is contained in [3-5].

2 LABORATORY TESTS

2.1 Test elements

Experimental tests were performed on four sets of beams. Each set consisted of three elements having the same dimensions (3300 x 150 x 250 mm). The synthetic summary of the test elements and their properties is presented in the form of a table (Table. 1).

Tab. 1. The summary of the test elements

Set	B-I	B-II	B-III	B-IV
Cross-section				
f_{cm} [MPa]	51.7	51.2	45.6	41.1
$f_{ctm,spl}$ [MPa]	3.58	3.21	3.03	2.79
E_{cm} [GPa]	30.3	29.6	28.5	30.0
f_{ym} [MPa]	563	563	548	555
E_s [GPa]	202	202	200	202
M_R [kNm]	26.96	27.00	26.62	53.29
M_{cr} [kNm]	7.35	6.18	5.87	5.60

where: ρ – tensioned reinforcement ratio, f_{cm} – mean compressive strength of concrete, $f_{ctm,spl}$ – mean tensile strength of concrete at splitting, E_{cm} – mean Young's modulus of concrete, f_{ym} – mean yield strength of reinforcing steel (longitudinal reinforcement), E_s – mean Young's modulus of reinforcing steel (longitudinal reinforcement), M_R – mean flexural load bearing capacity of the beam, M_{cr} – mean cracking moment.

2.2 Test procedure

The beams were loaded in a three-point bending test. During the study, deflection in the middle of the span and on the supports was recorded by means of inductive gauges with an accuracy of 0.001 mm. From the measurements, actual deflection was calculated at the centre of the span. After cracking of each beam, an inventory of cracks for at least five steps of load was performed. Due to the nature of calculations using its own numerical model, the focus was on the number and spacing of cracks. A detailed description of the tests is given in [6].

2.3 Measurement results

Deflections in the middle of the span recorded in the test are shown in the graphs (Figs. 1-4).

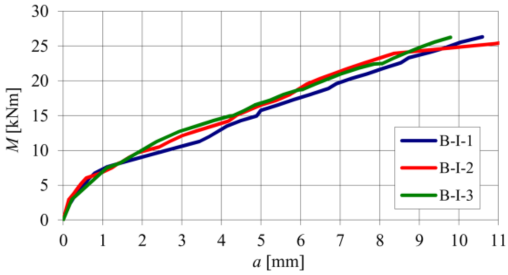


Fig. 1. Beam deflection in set B-I

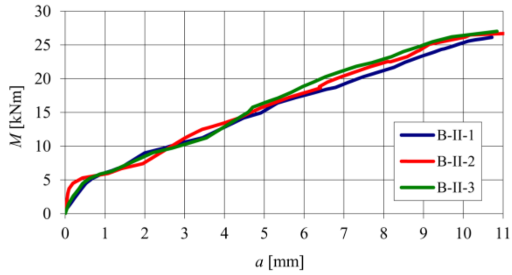


Fig. 2. Beam deflection in set B-II

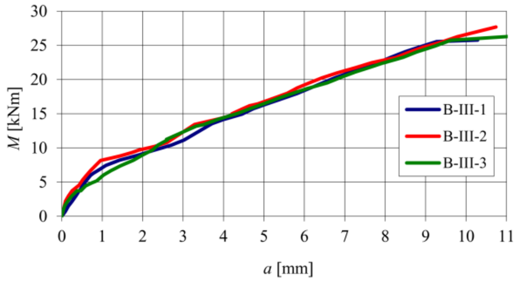


Fig. 3. Beam deflection in set B-III

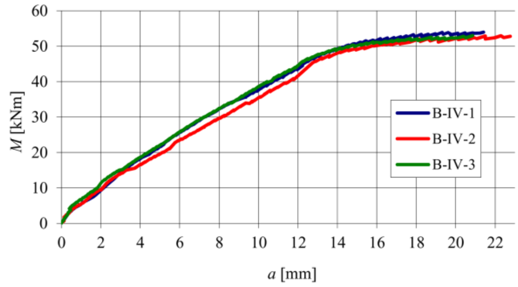


Fig. 4. Beam deflection in set B-IV

4 NUMERICAL CALCULATIONS, DISCUSSION OF THE RESULTS

Numerical calculations with the developed model were performed for selected load steps, at which cracks in the beam were drawn. The results are shown in the form of histograms for each of the beams (Figs. 5-8). The numerical model presented enabled the separation of deflections originating from flexural deformability of the element from crack effect. The following designations were applied: a_b – deflection dependent on the flexural deformability, a_{cr} – calculated deflection dependent on cracks, a_e – measured deflection.

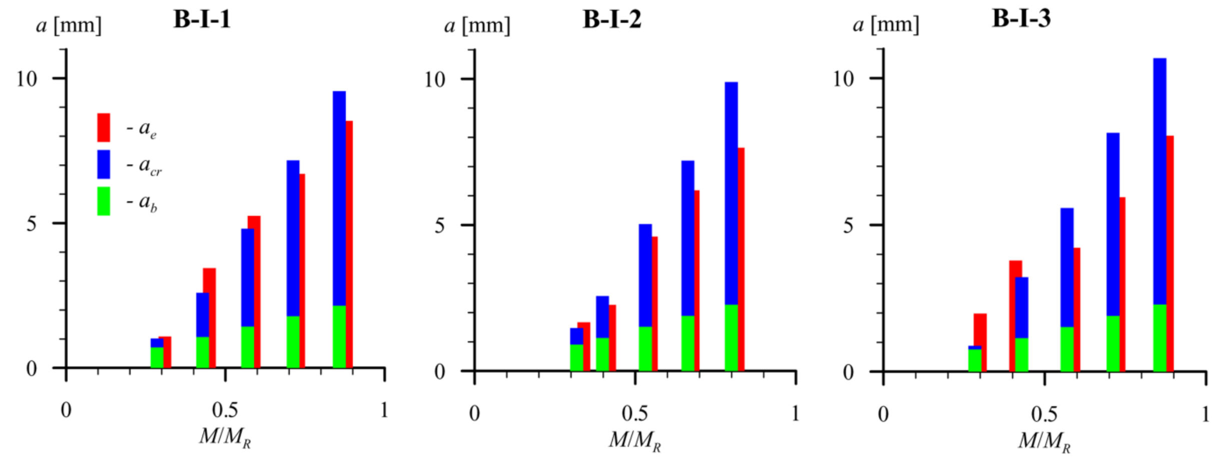


Fig. 5. Summary of results for set B-I

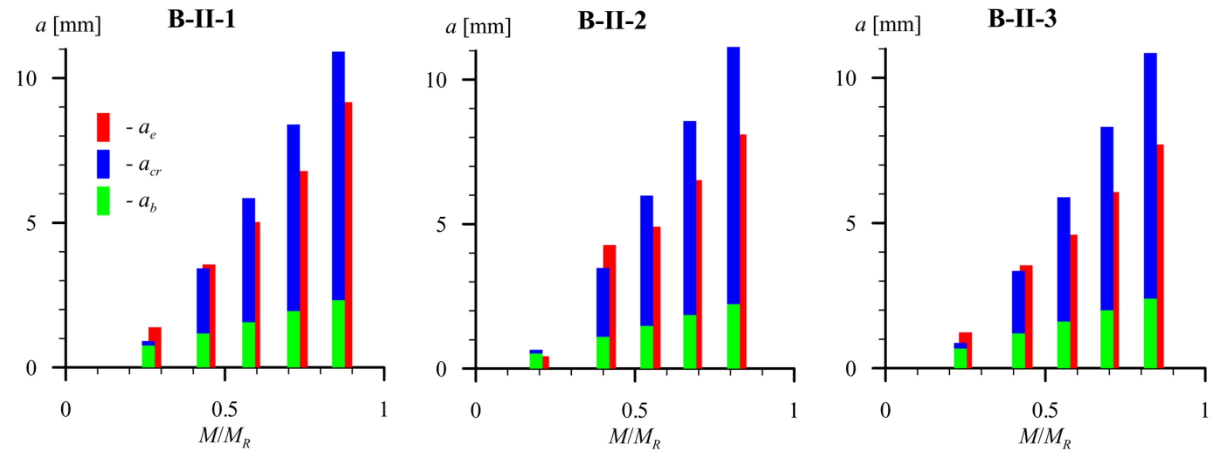


Fig. 6. Summary of results for set B-II

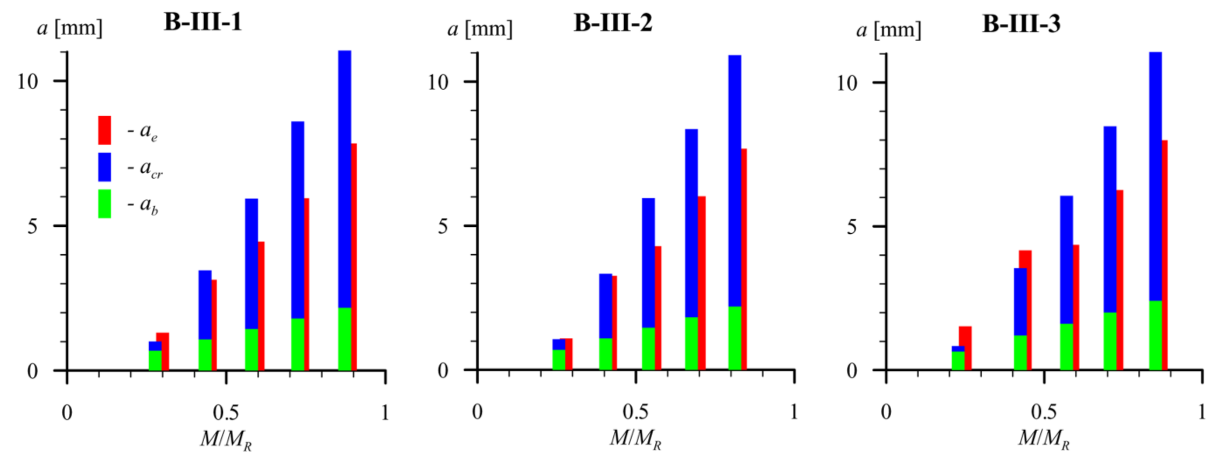


Fig. 7. Summary of results for set B-III

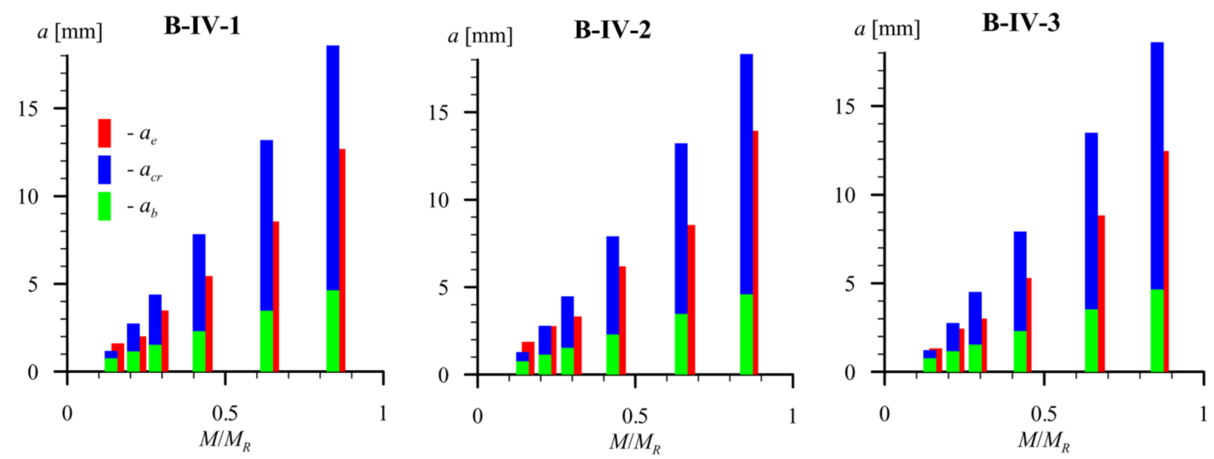


Fig. 8. Summary of results for set B-IV

The analysis performed lead to the following conclusions:

- deflections of elements caused by the flexural deformation of the element are multiplied in a quasi-linear manner over the range of loads in the case of elements less (sets BI, B-II, B-III) and more reinforced (set B-IV);

- along with an increase in the load, the share of the crack effect in deflection increases up to 75% (more reinforced elements) or 80% (less reinforced elements) in the final phase of the element operation;
- in the developed numerical model, the re-estimation of deflections takes place, it is greater the greater the severity of the load is.

Re-estimation of deflections has its base in the established theoretical model. It is assumed that the crack opens all the way up to the neutral axis. This is a simplification because the cracks formed in the final phase of the element's operation (closer to the supports) are subjected to a much smaller bending moment and they open to a less degree (they demonstrate lower rotational susceptibility).

In order to observe certain regularity, graphs (Figs. 9-12) are listed below. They include the impact of the global deflection multiplier δ as a function of the severity of the load. This factor allows the transition from the calculated deflection at to the actual (measured) deflection a_e, and it must be interpreted in accordance with the following relation (1).

$$a_t \cdot \delta = a_e . \tag{1}$$

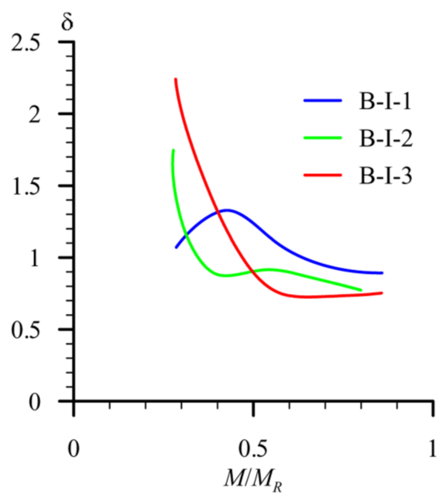


Fig. 9. Coefficient δ for beams in set B-I

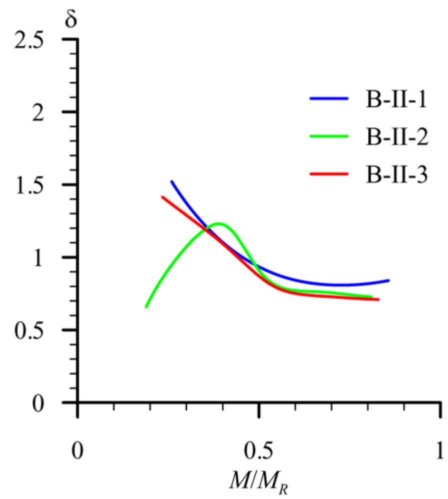


Fig. 10. Coefficient δ for beams in set B-II

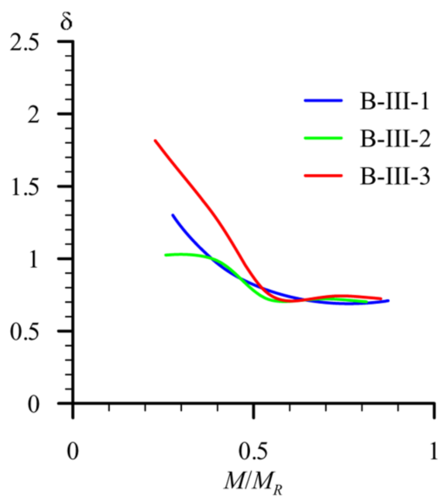


Fig. 11. Coefficient δ for beams in set B-III

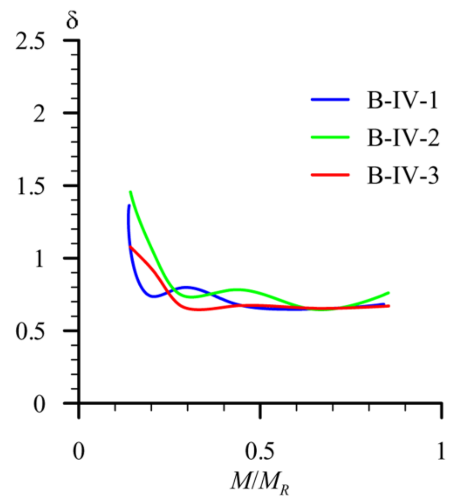


Fig. 12. Coefficient δ for beams in set B-IV

From these graphs, it can be seen that in the initial phases of the element operation, the coefficients are characterized by significant divergence. It is a consequence of the fact that the cracking process in these areas is very random and chaotic. An important observation is the fact that the re-estimation error stabilizes the ranges: 0.5-1.0 for less reinforced beams and 0.3-1.0 for more reinforced beams, and the coefficient δ is then approx. 0.78 and approx. 0.65, respectively (for other reinforcement degrees it is recommended to determine the coefficient from the linear interpolation). From an engineering point of view, these ranges are the most important, because rationally designed elements operate in these ranges. This allows us to consider the presented method to be effective, provided that the correction coefficient δ will be taken into account.

Another more advanced way to calibrate the presented method would be to modify the rotational susceptibility resulting from cracks. An additional reducing coefficient proportional, e.g. to the bending moment, should be applied.

4 SUMMARY

The paper presents a rigid finite element method for calculating deflections of reinforced concrete beams with cracks. Detailed analyses performed using own computational model showed its usefulness in civil engineering. The method of the calculation enabled the separation of part of the deflection resulting from the flexural deformability of the element and from the cracking phenomenon. The percentage of cracking in the total deflection at the high degree of effort reaches even 75-80%, depending on the degree of the beam reinforcement. The developed method produces a slight overestimation of the calculated deflections, which has its origin in the adopted model of the crack.

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